# APPLYING THE KALMAN FILTER TO THE EMITTER LOCATION PROBLEM USING AIRBORNE ANGLE-OF-ARRIVAL INFORMATION

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# THESIS

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by

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March 1973

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Applying the Kalman Filter to the Emitter Location Problem Using Airborne Angle-of-Arrival Information

by

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#### ABSTRACT

A scheme to locate emitter positions using post flight processing of discrete airborne emitter bearing angles-of-arrival information and recorded aircraft position coordinates by Kalman filter techniques is developed. The signal intercept system was assumed to be operating in a multi-emitter environment and all data was sampled at discrete but time varying intervals. The aircraft position data is filtered directly in latitude and longitude and emitter locations are computed in latitude and longitude using vector methods. An extended Kalman filtering scheme is developed to compute emitter coordinates directly in latitude and longitude and longitude coordinates.



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#### I. INTRODUCTION

With the ever increasing reliance of the military forces on electromagnetic systems for communication, reconnaissance, and weapons control, there is a correspondingly increasing requirement to detect and locate the position of the enemy emitters.

The complexity of this problem was shown by F. Pfendtner in [1]. When there are multiple emitters in reasonable proximity and several observations, called direction finding (DF) bearings, are made, the number of possible locations I, of these emitters is found by

$$I = m^2 n (n-1)/2$$

where m is the number of emitters and n is the number of observations.

If there were six emitters and ten observations of each emitter, there would be a possibility of 1,620 points of bearing line intersections, each a valid emitter position.

A scheme was developed in [1] to locate the most probable location of the emitter based on the mean of all points of intersection associated with a given target. This scheme did not provide the desired degree of accuracy. So, a scheme was developed to Kalman filter all of the DF bearing correlated with a given target, thus finding an optimal estimate of these observations. The first and last estimates



of the DF bearings to each target were used to determine emitter location by triangulation methods. This scheme provided solutions much closer to the desired accuracy.

Any solution is only as good as the data used to compute it, so, L. L. Coburn developed a data sort scheme in [2] which would compare all of the observations and correlate each data point with a specific target to avoid the possibility of basing the solution of an emitter position on observations not from that target. The sort was based on signal carrier frequency and pulse repetition frequency, PRF. The data sort utilized in [2] was only an initial sort in that any observations not passing a later comparison test were discarded and not saved for locating further emitter sites.

In this study an initial data sort based on signal carrier frequency and PRF is utilized, but all observations are eventually correlated to a target or are labeled as single line bearings.

Also for this report a routine was developed to determine optimal estimates of aircraft position coordinates by filtering and smoothing aircraft position observations.

An extended Kalman filter was developed to filter the estimate of the target position directly in latitude and longitude. This routine could be used to track moving targets, or to filter emitter positions based on data from several flights. Only signal parameters, the estimates of



aircraft position, observed DF bearing angles, time data, and the estimated target position would need to be stored from each flight for further processing.



### II. PROBLEM DESCRIPTION

#### A. KALMAN FILTER THEORY

Sequential estimation is characterized by the serial recursive processing of observations taken in time sequence. The result of every processing cycle is the current best estimate of the vector being estimated. This estimate therefore embodies all observation data up to and including the current observation. As a new observation is made, the current estimate is updated to reflect this most recent data.

In such an estimation scheme the calculations are identical in nature from cycle to cycle so they are ideally suited for implementation on a digital computer.

The problem presented in this report is that of the post flight processing of digitized data, but the program utilized in this case could easily be adapted for a real-time processor for in-flight computing and real-time locating of emitter sites.

The Kalman filter [3] is a recursive filter of the type needed for this application. Since the data inputs are already in digital form, the discrete form of the Kalman Filter are utilized for processing on a digital computer.

From the probabilistic description of the random signal and noise we can determine the probability with which a particular sample of the signal and noise will occur and we can therefore estimate x(k). This estimate will be denoted by  $\hat{x}(k)$ 



The discrete system under consideration satisfies

$$x(k+1) = \phi(k)x(k) + w(k)$$
 (1)

$$z(k) = h(k)x(k) + v(k)$$
(2)

where x is an n x 1 state vector, z is an m x 1 output vector, w is a zero-mean n x 1 vector of state excitation white noise, uncorrelated with the zero-mean-additive observation white noise vector  $\mathbf{v}$ ,  $\phi$  is the state transition matrix and h is the observation matrix. The noise statistics are

$$E\left[v(k) \ v(j)^{T}\right] = R(k) \ \delta(k,j)$$
 (3)

$$E\left[w(k) \ w(j)^{T}\right] = Q(k) \ \delta(k,j) \tag{4}$$

$$E\left[v(k) \ w(j)^{T}\right] = 0 \quad \text{for all } (k,j)$$
 (5)

$$\delta(k,j) = \begin{cases} 0 & k \neq j \\ 1 & k = j \end{cases}$$
(6)

The Kalman filter recursion equations [4] are summarized below where  $\hat{X}(k|j)$  denotes the estimate of the state X(k) based upon j measurement observations  $Z(1), Z(2), \ldots, Z(j)$ .

$$P(k|k-1) = \phi(k,k-1)P(k-1|k-1)\phi(k,k-1)^{T} + Q(k)$$
 (7)

$$G(k) = P(k|k-1)H(k)\left[H(k)P(k|k-1)H(k)^{T}+R(k)\right]^{-1}$$
 (8)

$$P(k|k) = P(k|k-1) - G(k)H(k)P(k|k-1)$$
 (9)

$$\hat{X}(k|k) = \hat{X}(k|k-1) + G(k) \left[ Z(k) - H(k) \hat{X}(k|k-1) \right]$$
 (10)

$$\hat{X}(k|k-1) = \phi(k,k-1)\hat{X}(k-1|k-1)$$
(11)



where G(k) is the Kalman filter gain matrix and P(k|j) is the error covariance matrix.

#### B. SYSTEM MODELS

The emitter position locating system modeled was divided into three distinct systems, (1) aircraft navigation system, (2) the DF bearings, and (3) the target location. The combined system consists of five linear Kalman filters, plus one linearized Kalman filter, called an extended Kalman filter, and two smoothing filters. Each of these models will be described briefly in this section with more detailed descriptions in Section III, "Computational Procedures."

All of the systems are modeled by a linear  $1/s^2$  plant. The state transition matrix,  $\phi$ , is

$$\phi(k+1,k) = \begin{bmatrix} 1 & T(k+1) \\ 0 & 1 \end{bmatrix}$$
 (12)

where T(k+1) = TIME T(k+1) - TIME T(k), TIME T being the time of observation.

To minimize the affects of the inherent uncertainty of the system model, the values of Q should be increased to a value which ensures that each observation is utilized. The effect of increasing the magnitude of Q is seen from (7) to be larger values of the error covariance matrix, P, which leads to an increase in the filter gain matrix G. This means that the filter is paying more attention to the actual measurement to compensate for errors in the plant model.

Thus, the more nonlinear the system dynamics, the larger Q



should be. If the measurement errors are in question, R should be increased which, as can be seen from (8), will decrease G so the filter relies more on the total effect of all previous measurements, i.e. the previous estimate of the system states [5].

# 1. Aircraft Navigation System Model

The navigation system is modeled by a constant velocity plant for each direction of aircraft travel, east-west and north-south, with the system states for each plant being aircraft position and aircraft velocity. Only the aircraft position fixes were observable so the objective of this filter is to filter the noisy observed position coordinates to obtain optimal estimates of own aircraft location. These estimates are used in processing the DF bearings and target positions.

Since only aircraft position coordinates were observed, the observation matrix is  $H = [1 \ 0]$ . This made W(k), V(k) and R(k) scalar variance terms and the bracketed terms in (8) and (10) become scalar as well.

Since W is a scalar, (4) becomes

$$\Gamma E \left[ W W^{T} \right] \Gamma^{T} | = \Gamma \Gamma^{T} E \left[ W^{2} \right]. \tag{13}$$

Substituting the expression for I into (13) yields

$$Q(k) = E[W^{2}]\begin{bmatrix} \frac{T^{4}(k)}{4} & \frac{T^{3}(k)}{2} \\ \frac{T^{3}(k)}{2} & T^{2}(k) \end{bmatrix}$$
(14)



# 2. DF Bearing Model

The system states for the problem of angle filtering are DF bearing angle-of-arrival and bearing rate. In this program the system dynamics are also modeled by a constant velocity plant. The bearing rate is unobservable so the observation matrix is  $H = \begin{bmatrix} 1 & 0 \end{bmatrix}$  as in the navigation data filter equations.

This model does not accurately represent the actual system since the estimated angular rate error increases with time[1]. However by introducing a non zero Q matrix this error can be reduced. The effect of Q on the accuracy of this model can be seen in the results obtained by L. L. Coburn [2].

# 3. Target Position Coordinates Model

The states of the target location system are latitude, latitude rate, longitude, and longitude rate while the observations are the noisy DF bearings only. To compute the optimal estimate of the states, the system was modeled by two linear constant velocity systems, one each for latitude and longitude. A linearized transformation of the measurement equation was utilized to provide DF bearing information to the latitude and longitude filters in a form they could use to filter their states.

The models for the two linear systems are again  $1/s^2$  plants with H = [1 0].

The target location system model is shown in Fig. 1, and is discussed further in Section III, part F, "Extended Kalman Filter."



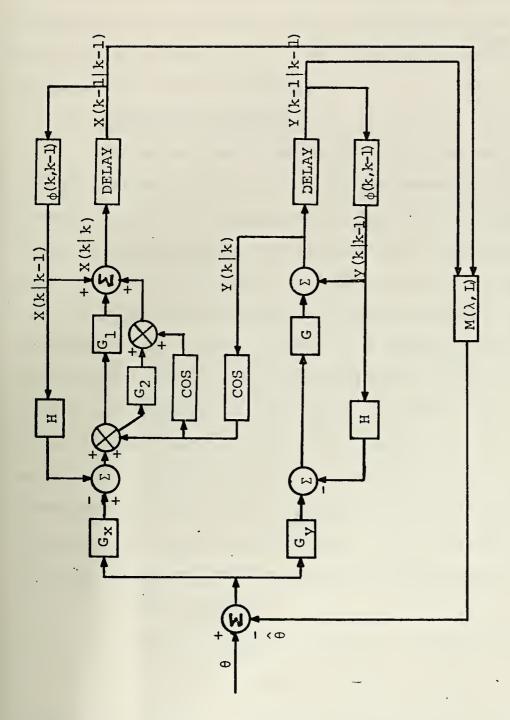


Figure 1. Extended Kalman Filter Block Diagram.



#### C. SMOOTHING

Two types of smoothing are utilized in this program, fixed point with the angle filter and fixed interval with the navigation data filter. The fixed point smoothing technique will be discussed first.

# 1. Fixed Point Smoothing

In the process of angle filtering, the estimate of the DF bearing,  $\hat{\theta}(n|n)$ , is a weighted "average" of all previous noisy DF bearings,  $\theta(1),\theta(2),\ldots,\theta(n)$ . However the actual position finding algorithm (POSIT) utilizes the estimate of the first DF bearing,  $\hat{\theta}(1|n)$ , as well as  $\hat{\theta}(n|n)$ . So, the first bearing should be somehow weighted by all successive cuts,  $\theta(2),\ldots,\theta(n)$ . The technique used to accomplish this filtering is called fixed point smoothing. The smoothing equations are similar to the Kalman filter equations in that they are recursive in nature and functions of similar statistical parameters. The difference is that the smoothing filters produce a previous estimate of the state subject to successive measurements. The fixed point smoothing equations used in this program are summarized below [6].

# a. Gain Equation

$$\hat{X}(k|j) = \hat{X}(k|j-1) + W(j)H \quad (j)^{T}R^{-1}(j) \left[Z(j) - H(j)\phi(j,j-1)\hat{X}(j-1|j-1)\right]$$
(15)

where j = k+1, k+2,..., and the initial condition is  $\hat{X}(k|k)$ .



b. Gain Equation

$$W(j) = W(j-1)\phi (j,j-1)^{T} [I-S(j)P(j|j)]$$
 (16)

where W(k) = P(k|k) and  $S(j) = H(j)^{T}R^{-1}(j)H(j)$ 

Covariance Equation

$$P(k|j) = P(k|j-1) - W(j) \left[S(j)P(j|j-1)S(j) + S(j)\right] W(j)^{T}$$

$$(17)$$

where the initial condition is P(k|k). P(j|j) and P(j|j-1) are the error covariance matrices from the optimal filter.

# 2. Fixed Interval Smoothing

The second type of smoothing utilized is the fixed interval smoothing. In the navigation data filtering routine, the flight track is broken into legs with a new leg being initiated any time the aircraft flies for more than two minutes without recording a position fix. Each leg is then an interval and each interval is filtered and smoothed separately. The motivation for the fixed interval smoothing routine is similar to that for smoothing in the angle filtering routine, i.e. each estimate should be influenced by each successive measurement as well as by each previous one. The fixed interval smoothing equations utilized in the angle filter are [6]. Utilized in the angle filter are [6]:

a. Filter Equation

$$\hat{X}(k|n) = \hat{X}(k|k) + A(k) \left[ \hat{X}(k+1|n) - \hat{X}(k+1|k) \right]$$
 (18)

for k = n-1, n-2, ..., 0, where X(n|n) is the boundary condition for k = n-1.



b. Gain Equation

$$A(k) = P(k|k) \phi (k+1,k)^{T} P^{-1}(k+1|k)$$
 (19)

c. Covariance Equation

$$P(k|n) = P(k|k) + A(k) [P(k+1|n) - P(k+1|k)] A(k)^{T}$$
 (20)

This filter has the advantage taht it incorporates the covariance terms, P(k|k) and P(k+1|k), as well as the estimates X(k|k) and X(k+1|k) from the Kalman filter equations, so the required number of computations is minimized. Since k = n-1, n-2,...,0, it is clear that this system of equations is recursive backwards in time so it does indeed produce a smoothed estimate of each navigational fix subject to all successive fixes on that leg.

### D. INITIAL EMITTER POSITION FIXING

Each position on the earth can be considered to be the tip of a vector whose length is the radius of the earth.

In this problem the radius of the earth is assumed to be constant. Even though it is known that the constant radius assumption is an inaccurate one, using a constant value for the radius produces an error of less than two percent of the distance involved anywhere on the surface of the earth [7]. Also by using this assumption this method is equally valid over the entire globe without regard for the radius.

The position vector can be described either by its latitude and longitude or in x,y,z coordinates on a three



dimensional coordinate system with the center of the earth as the origin of the system, as in Fig. 2.

The latitude is the angle in a meridian plane measured from the equatorial plane to the radius vector of the aircraft. Longitude is the angle in the equatorial plane measured from the Greenwich meridian to the meridian of the aircraft. The latitude of a point on the earth is in the range 0 to 90 degrees in the northern hemisphere and from 0 to -90 degrees in the southern hemisphere. The longitude of a point on the earth is in the range 0 to 180 degrees in the eastern hemisphere and from 0 to -180 in the western hemisphere. In the 3-D right hand coordinate system the positive z axis passes through the geographic north pole, the positive x axis passes through the Greenwich meridian and lies in the equatorial plane and the positive y axis passes through 90 degrees east longitude and lies in the equatorial plane [8].

The relationship between the latitude and longitude coordinates  $(\phi,\theta)$  and the (x,y,z) coordinates is expressed by the formulae

$$\mathbf{x} = \mathbf{cos}\phi \ \mathbf{cos}\theta \tag{21}$$

$$y = \cos\phi \sin\theta \tag{22}$$

$$z = \sin \phi \tag{23}$$

From vector algebra we know that when two vectors are multiplied together vectorially the resultant is called the cross product. The resultant is another vector whose direction is perpendicular to the plane of both of the original



Figure 2. Illustration of Relation Between Station  $(\phi,\theta)$  and (x,y,z) coordinates.



vectors and is called the normal vector. Therefore if the aircraft position vector is crossed into the vector normal to the DF bearing plane, a normal vector parallel to the surface of the earth is produced. This multiplication is carried out for the smoothed initial DF bearing,  $\hat{\theta}(1|n)$ , and the filtered last bearing,  $\hat{\theta}(n|n)$ . Then if the bearing vectors are assumed parallel to the surface of the earth at the target position, their cross product will produce a vector through the center of the earth at their point of intersection. This vector is the position vector of the target.

The assumption of the DF bearing being parallel to the earth's surface at the target and at the position of the aircraft can be easily understood when it is realized that the bearing vector is actually the projection of the plane containing the DF bearing onto the surface of the earth.

The difficulty of the vector method arises when an attempt is made to describe the bearing vector in an earth centered cartesian coordinate system so the cross product can be obtained. This description is found by a series of three coordinate system rotations. The resultant of a succession of coordinate system rotations is simply the dot product of the coefficient matrices of each rotation [1] and [9].

The rotations made in this program are shown in Figs. 3, 4, and 5. The original coordinate system is denoted (i,j,k), after the first rotation (i\*,j\*,k\*), after the second



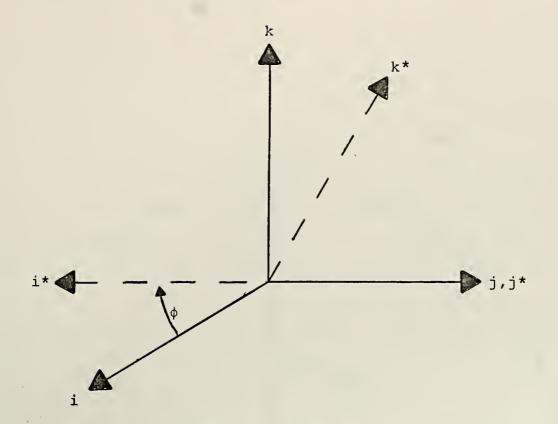


Figure 3. Rotation of Axes about j Axis.

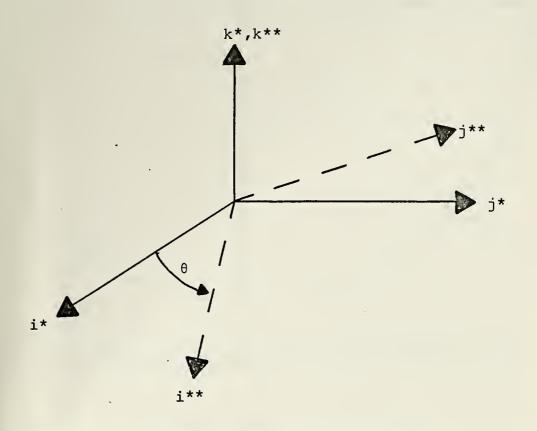


Figure 4. Rotation of Axes about k\* Axis.

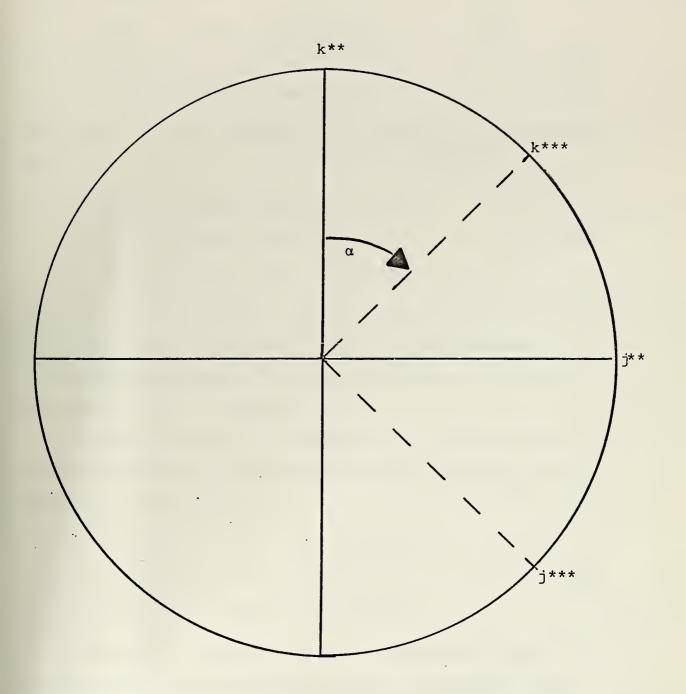


Figure 5. Rotation of Axes About Local Vertical.



rotation (i\*\*,j\*\*,k\*\*), and after the third rotation (i\*\*\*,j\*\*\*,k\*\*\*). The first rotation about the negative j axis is expressed as

$$\begin{bmatrix} i^* \\ j^* \\ k^* \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} i \\ j \\ k \end{bmatrix}$$
 (24)

The second rotation is about the k\* axis and is represented by

$$\begin{bmatrix} i^* * \\ j^* * \\ k^* * \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i^* \\ j^* \\ k^* \end{bmatrix}$$
(25)

The resultant of the first two rotations is a target centered coordinate system expressed in cartesian coordinates. In this coordinate frame, i\*\* is local vertical, j\*\* is local west and k\*\* is local north.

The third rotation is a rotation by  $-\alpha$  about the i\*\* axis (local vertical) which after making simplifying trig-onometric substitutions is

$$\begin{bmatrix} i^* * * \\ j^* * * \\ k^* * * \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\alpha & \cos\alpha \\ 0 & -\cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} i^* * \\ j^* * \\ k^* * \end{bmatrix}$$
(26)

Since k\*\*\* is the unit vector in the bearing plane, j\*\*\*
is the normal to the bearing plane and after carrying out
the indicated matrix multiplications, is expressed in terms
of the original coordinates as



$$N = j^{***} = (-\sin\alpha\sin\theta - \cos\alpha\sin\phi\cos\theta)i$$

$$+ (\sin\alpha\cos\theta - \cos\alpha\sin\phi\sin\theta)j$$

$$+ (\cos\alpha\cos\phi)k. \tag{27}$$

Then by cross multiplying the aircraft position vector associated with N, into N, the vector which represents the DF bearing vector is determined. This method was utilized in developing subroutine POSIT which is discussed in Appendix C.

#### E. EXTENDED KALMAN FILTER

Since the Kalman filter is optimum only when the system differential equations and measurements are linear, a relationship had to be found to linearize the nonlinear measurements associated with emitter locating. The sought for relationship can be obtained by a Taylor series expansion of the target location. This series expansion of the nonlinear measurements can be substituted for the coefficient matrices in the Kalman filter recursive gain equations and computation then proceeds just as in the discrete algorithm as described previously in this report. This employment of the Kalman filter is frequently referred to as the "Extended Kalman Filter." It is an intuitive but frequently successful application of the Kalman filter in the absence of truly optimum filters for non-linear systems [10].

These techniques are only approximate. They require that the disturbances, measurement noises, and uncertainties in the state be of such a size that the higher-order terms



ignored in computing the error are insignificant. It was felt that for this system, where one of the objectives was simplicity of computation, that though the higher order terms are existent the system dynamics model used was a satisfactory approximation of the actual plant dynamics.

In the case of emitter locating from an aircraft using the extended Kalman filter routine, we are actually filtering in latitude and longitude while our observations are noisy DF bearings only, i.e.

$$Z(k) = \theta(k) + V(k). \tag{28}$$

It is obvious that no pseudo-cartesian coordinates can be generated from this measurement. The non-linear measurement process must therefore be represented by a linear approximation.

From the development of the Kalman filter equations [6], it can be seen that

$$\hat{X}(k|k) = \phi(k,k-1) \hat{X}(k-1|k-1) + E[\hat{X}(k|k)|\hat{Z}(k)]$$
 (29)

Let  $G(k)\hat{Z}(k) = E[\hat{X}(k|k)\hat{Z}(k)]$  where  $\hat{Z}(k) = M(k)\hat{X}(k|k-1)$ . So the estimated measurement equation becomes

$$\hat{\mathbf{Z}}(\mathbf{k}) = \hat{\boldsymbol{\theta}}(\mathbf{k}) \tag{30}$$

where [2]

$$\hat{\theta}(k) = TAN^{-1} \left( \frac{(\lambda_T - \lambda) \cos L_T}{L_T - L} \right)$$
(31)

where  $\lambda_{\rm T}$  is the target longitude,  $\lambda$  is the aircraft longitude,  $L_{\rm T}$  is the target latitude, and L is the aircraft latitude.



Substituting this expression into the Kalman filter equations, the recursive filter equation becomes

$$\hat{X}(k+1) = \phi(k+1,k)\hat{X}(k) + G(k)[\theta(k) - \hat{\theta}(k)].$$
 (32)

Expanding the measurement equation

$$\hat{\theta} = M(\lambda, L) \tag{33}$$

about the most recent optimal estimate and ignoring higher order terms results in

$$\hat{\theta} = M(\lambda^*, L^*) + \frac{\partial \hat{\theta}}{\partial \lambda} \left| \lambda^* + \frac{\partial \hat{\theta}}{\partial L} \right| L^*.$$
 (34)

from which we see that

$$M = \begin{bmatrix} \frac{\partial \hat{\theta}}{\partial \lambda} & \frac{\partial \hat{\theta}}{\partial L} \end{bmatrix}$$
 (35)

The Kalman filter recursion equations, (7) through (11), may then be rewritten to include the nonlinear observation matrix M, and are given by

$$P(k|k-1) = \phi(k,k-1)P(k-1|k-1)\phi(k,k-1)^{T} + Q(k)$$
 (36)

$$G(k) = P(k|k-1)M(k) \left[M(k)P(k|k-1)M(k)^{T} + R(k)\right]^{-1} (37)$$

$$P(k|k) = P(k|k-1) - G(k)M(k)P(k|k-1)$$
 (38)

$$\hat{X}(k|k) = \hat{X}(k|k-1) + G(k) \left[ Z(k) - M(k) \hat{X}(k|k-1) \right]$$
 (39)

$$\hat{X}(k|k-1) = \phi(k,k-1)\hat{X}(k-1|k-1)$$
(40)



## III. COMPUTATIONAL PROCEDURES

## A. NAVIGATIONAL DATA FILTERING - (SUBROUTINE NAV)

The first step in the problem of finding a target is that of filtering the aircraft position estimates. Aircraft navigation data was sampled and recorded at discrete but time varying intervals. As stated in Section II, part B.1, only the position fixes were measured so the velocity is only estimated and is strictly a product of the model used to represent the aircraft. Since the aircraft position fixes were taken at random intervals, there are extended periods during which no navigation data is available and when the flight track could be altered drastically. Therefore, the flight track is broken into legs to ensure that the program does not attempt to filter through these abrupt changes.

The criteria used in this program to establish the end of a leg is a simple time test. If T(k), the time between navigation fixes, exceeds 120 seconds, the current leg is terminated and a new leg is initiated.

If a leg was found to consist of three or fewer data points, that leg was neither filtered nor smoothed, but rather the measured data was used for the filtered and smoothed estimate of position, i.e., SLADSM(K)=ACLAD(K). No estimation of velocity was computed for these legs. After the entire flight track was divided into legs, the Kalman filter was initialized.



The navigation data filter consists of two nearly identical Kalman filters, one to filter latitude and the other for longitude filtering. The system dynamics for the aircraft are modeled in each direction by a constant velocity plant.

Since equal angles of latitude and longitude yield equal distances of movement only at the equator, a correction factor must be applied to angular measure as either pole is approached from the equator.

At 60 north latitude, for example, one degree of movement in latitude measures sixty nautical miles while one degree of movement in longitude only measures thirty nautical miles. This requires that the longitude angular distance be corrected by the cosine of the local latitude coordinate so that the two filters will have equal units of distance. The position estimates of the navigation data filters are based on estimated distances traveled, i.e.

$$\hat{X}l(k|k) = \hat{X}l(k|k-1) + \hat{X}l(k|k-1) T(k)$$
 (41)

The estimated changes in longitude position will be off by a value of 1/cos(latitude), so (41) was modified to

$$\hat{X1}(k|k) = \hat{X1}(k|k-1) + \hat{X1}(k|k-1) T(k)\cos(latitude) (42)$$

Equivalently,  $\hat{X1}(k|k-1)$  can be reduced by the cosine of the latitude directly, which is the method used in this program. The equation used in NAV then is

$$VELED(K) = (VELED(KK) + G2(K)*ELON(K))*COS(SLA(K))$$
 (43)



where VELED is the aircraft velocity east in degrees per second, G2 is the Kalman filter gain, ELON is the Kalman filter error term and SLA is the latitude coordinate of the aircraft. The same correction is applied to the smoothed  $\hat{x}$ l(k) term, VELEDS, in the smoothing filter.

To initialize the latitude filter, the initial position estimate, SLAD(KI), was set equal to the first measured position, ACLAD(KI). The aircraft velocity was not observable, so the initial velocity was estimated by

$$VELND(KI) = (ACLAD(KI+1) - ACLAD(KI))/T(KI+1)$$
 (44)

where T(KI+1) is the time increment from time KI to time KI+1.  $\hat{X}(1|0)$  was assumed equal to  $\hat{X}(0|0)$  so SLATD(KI), the predicted value of SLAD, was also set equal to ACLAD(KI).

The longitude filter was initialized in the same manner as the latitude filter with the only change being that the velocity east was corrected by the cosine of the latitude to correct for the curvature of the earth as discussed above. Therefore

$$VELED(KI) = ((ACLOD(KI+1) - ACLOD(KI)/T(KI+1))$$

$$X COS(SLA(KI))$$
(45)

The initial uncertainty of position and velocity on filter initialization was accounted for in the initial values of the error covariance matrix P(1|0). The error in the initial position fixes by the navigational computer were assumed to be very small and so the initial covariance matrix P(1|0), was set equal to the identity matrix.



The measurement noise, v, is assumed scalar so by (3)

$$E\left[vv^{T}\right] = R. \tag{46}$$

The value of R will change for each system, depending upon the accuracy of that system, but it is assumed constant for each data set.

The smoothing filter is recursive in negative time so the first estimate of position computed by the smoothing filter is the next to the last fix in that leg. Therefore to initialize the smoothing filter, the first smoothed estimate SLADSM(N), was set equal to the last filtered position estimate SLAD(N). The error covariance matrix P, used to compute the smoothed position estimates is the same matrix computed for the Kalman filter, so there was no P matrix to initialize.

It was found that occasionally the aircraft navigational computer would produce a totally erroneous position fix, to check for this occurrence, each Kalman filter error term,  $\hat{E}(k) = Z(k) - \hat{X}(k+1|k), \text{ was checked and if it exceeded a certain value, denoted test, the data point was rejected.}$  The estimate of the state,  $\hat{X}(k+1|k)$ , was inserted as the estimate of position, i.e. in the latitude filter

$$SLAD(K) = SLAD(K-1) + VELN(K-1)T(K)$$
(47)

The equations used in the navigation data filter (SUBROUTINE NAV) are derived in Appendix A.



#### B. ANGLE FILTERING - (SUBROUTINE GEORGE)

After the navigation data is filtered and smoothed the processing switches to subroutine GEORGE, which is basically the program presented in [2]. This subroutine sorts the emitter data and then filters and smooths the DF bearings.

To adapt the program presented in [2], for use in this problem, the data sort routine was changed, a new smoothing filter was inserted and the emitter position locating algorithm was replaced by a vector solution method which is included in subroutine POSIT.

The data sort used in this program is shorter than the original routine. In the data sort routine, the error interval associated with the measured values of signal frequency and PRF was opened up to approximate the errors of the system being simulated. This test interval can easily be changed to any desired width to fit the system being simulated.

It was found that the PRF value was zero in some data sets. In these cases, the DF bearing associated with these particular points were correlated with the previous signal with the same carrier frequency.

All of the DF bearing angles are checked for target correlation as in [2], but instead of discarding any data points which do not pass the test  $E^2(K) < (P11(K) + RCUT)*TSTCUT$ , being discarded, they are assigned to a new target. RCUT is the assumed value of the variance of the measurement noise v, and TSTCUT is a multiplier to vary the



size of the gate. This routine was inserted because with the method used in [2], only one target of each FREQ and PRF could be processed from each data set; data from all other similar emitters were discarded.

The smoothing routine utilized here is shorter and does not require the computation of the inverse of the P matrix as was required in [2], since computing the inverse of a matrix consumes a lot of computer time. The smoothing filter equations used are derived in Appendix B.

After each DF bearing, THTD(K), is filtered and the first DF bearing, THTD1(k), is smoothed based on all successive bearing estimates, a test is made to determine if processing should be switched to extended Kalman filtering, which is done in subroutine EXTEND. This test compares P11(K) with the product of EXTEST and RCUT, where EXTEST is a multiplier to vary the size of the gate. If EXTEST is set equal to zero, the processing will always stay in the angle filter. As EXTEST is increased the processing will switch to EXTEND earlier and earlier in the data sequence but the initial estimate of the target position used to initialize the extended filter will be worse, as will the initial value of the P matrix in the extended filter equations described in part F of this section.

If it is decided that all of the DF bearings correlated to a target will be angle filtered, GEORGE calls subroutine PREPARE which in turn calls subroutine POSIT which computes the emitter location based on the DF bearings THTDl and



THTD directly. These subroutines are discussed later in this section.

The value of the multipliers of both tests in GEORGE,

TSTCUT and EXTEST, are calling arguments for the subroutine
so they can be controlled by the main program.

All of the Kalman filter initializations for GEORGE are discussed in [2]. The fixed delay smoothing equations are initialized with the initial smoothed estimate THTD1(1), being set equal to the initial filtered estimate THTD(1). THTD(1) was set equal to the measured value of the first DF bearing, THETAD(KI).

#### C. SUBROUTINE PREPARE

When vector products are used, the order of multiplication is critical; if the order is reversed, the resultant will be reversed. In the problem of emitter position locating, a reversed resultant will produce a fix on the opposite side of the earth. PREPARE is written to prevent this occurrence. First the direction of the track of the aircraft is computed. Then THTD and THTDl are checked to determine on which side of the track they lie and ensure that they both lie on the same side of the track. If they do not, no position fix is calculated. If they do lie on the same side of the track, they are checked to see if they cross. If not, no fix is computed. PREPARE then computes the calling arguments for POSIT - ACLAD, ACLON, and THD. ACLAD and ACLON are the latitude and longitude coordinates respectively of the aircraft at the times the two DF bearings



were recorded. THD contains the direction of the two bearings THTD1 and THTD. In so doing, it ensures that the DF bearing with the larger angle of arrival is crossed into the bearing with the smaller angle of arrival and not vice versa. PREPARE is also called by ELIPS6 and EXTEND.

# D. COMPUTING INITIAL POSITION - (SUBROUTINE POSIT)

POSIT then computes the desired location by the method described in Section II, part D. First the bearing vectors,  $\hat{\theta}(1|n)$ , and  $\hat{\theta}(n|n)$ , are expressed in the earth centered cartesian coordinate system. Then the cross product of the two bearing vectors is computed giving the target vector in x, y, z coordinates. These coordinates are converted to latitude and longitude coordinates by

$$TLAD = TAN^{-1} \left( \frac{X3}{\sqrt{X1^2 + X2^2}} \right)$$
 (48)

$$TLOD = TAN^{-1} \left( \frac{X2}{X1} \right)$$
 (49)

where X1, X2, and X3 are the x, y, z coordinates respectively.

An example illustrating the method used to find the target

vector is given in Appendix C.

#### E. DEFINING THE ERROR ELLIPSE - (SUBROUTINE POINTS)

If it is decided that the data should be processed by the extended Kalman filter, POINTS is first called. POINTS uses POSIT to compute the intersections of the edges of the cones of error associated with THTD and THTD1. These points are shown in Fig. 6. POINTS ensures that PTLAT(1,I,J),



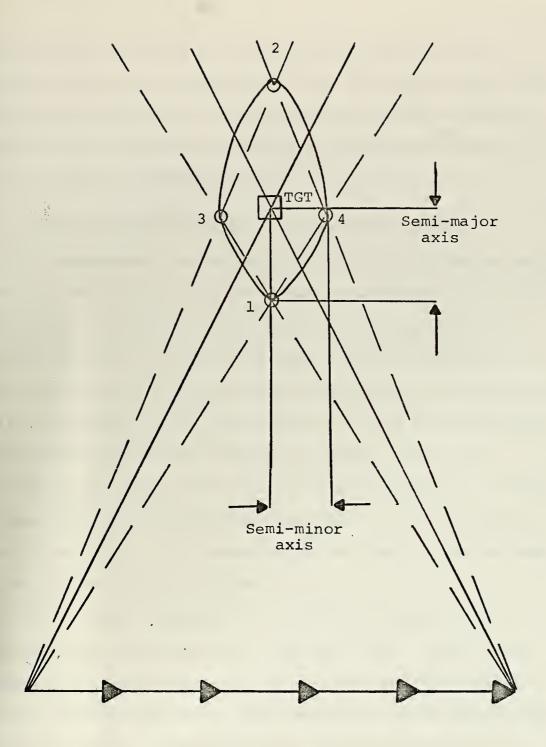


Figure 6. Target Position Error Covariance Ellipse.



PTLON(1,I,J) is always the intersection closest to the flight track and it determines if the DF bearings THTD1 and THTD do in fact cross. If they do not, it makes sure that EXTEND is not called by returning to the calling program at a point after the EXTEND call.

# F. EXTENDED KALMAN FILTERING - (SUBROUTINE EXTEND)

In linearizing the system by using a Taylor series expansion, it was assumed that the M matrix would be fairly constant over the range of uncertainty of x, therefore the initial value of x,  $\hat{x}(0)$ , must be a good approximation of the true value of x or the filter can diverge. To find an initial estimate of the true value of x, the DF bearings are first angle filtered in subroutine GEORGE. When the covariance of error of the current bearing estimate, Pll(K), becomes less than EXTEST times the variance of the measurement noise, RCUT, the processing is switched to the extended Kalman filter routine, EXTEND.

EXTEND calls PREPARE for an initial estimate of the target position coordinates, TLAD, and TLOD. EXTEND then computes the semi-major axis, a, and the semi-minor axis, c, of the error covariance ellipse described by the coordinates from POINTS and centered at the coordinates computed by PREPARE.

The length of the semi-major axis of the error ellipse was assumed to be the distance from the initial target location, TGT, to the intersection of the error cones closest to the flight track. These points are shown in Fig. 6.



The length of the semi-minor axis is found in the following manner.

The distance from the target position, TGT, to point 3 and the distance from TGT to point 4 are computed. The projection of the average of these two distances on to a line perpendicular to the semi-major axis is the length of the semi-minor axis.

The orientation of the error ellipse on the surface of the earth is taken to be the amount of clockwise rotation of the semi-major axis from the meridian passing through the center of the ellipse.

A and c uniquely describe an ellipse in an x'y' coordinate frame according to

$$\frac{x^2}{c^2} + \frac{y^2}{a^2} = 1. {(50)}$$

The x'y' system is rotated by an angle  $(90-\alpha)$  degrees from the xy system in which the extended Kalman filter functions. The x and y axes correspond to longitude and latitude respectively on a flat earth model. The extended filter will find an optimal estimate of the coordinates of the emitter position in the xy system. To describe the error ellipse in the xy system, a coordinate system rotation was made according to the transformation matrix.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$
 (51)



After performing this transformation and making simplifying trigonometric substitutions (see Appendix D) the initial values for the covariance matrix P, become

$$P11(KI) = (A \sin\alpha)^2 + (C \cos\alpha)^2$$
 (52)

$$P12(KI) = (A^2 - C^2) \sin\alpha \cos\alpha$$
 (53)

$$P22(KI) = (A \cos\alpha)^2 + (C \sin\alpha)^2$$
 (54)

Using (31), EXTEND computes an initial estimate of  $\hat{\theta}$ , TX, which is used to compute the initial extended filter error term, ER(KI)=THETA(KI)-TX(KI), where THETA is the measured DF bearing angle in radians. The initial gains associated with the nonlinear measurement equation are found from (37). The initial estimate of longitude, XTD, is set equal to TLOD, and YTD, the initial value of latitude, is set equal to TLAD.

The initial inputs to the linear longitude and latitude filter, XTDl and YTDl, are then found by substituting (11) into (10), rewritten as

$$XTD1(KI) = XTD(KI) + GX(KI)ER(KI)$$
 (55)

$$YTD1(KI) = YTD(KI) + GY(KI)ER(KI).$$
 (56)

Then the outputs, XTD and YTD, of the linear filters are fed into the linearized filter and the process proceeds as described for initialization.

The longitude and latitude filter equations are identical to the corresponding filters in subroutine NAV except for notation, as listed in Appendix A.



The  $\Phi$  matrix for the linearized measurement filter is the identity matrix since the states of this filter are longitude and latitude only.

On each recursion of the filter the error term ER, is compared with a gate identical to the correlation gate in subroutine GEORGE. Any DF bearings failing the test are assigned to a new target and are processed either in the angle filter or in EXTEND on a later call of the subroutine. The extended Kalman filter equations are derived in Appendix D.



# IV. PRESENTATION OF RESULTS

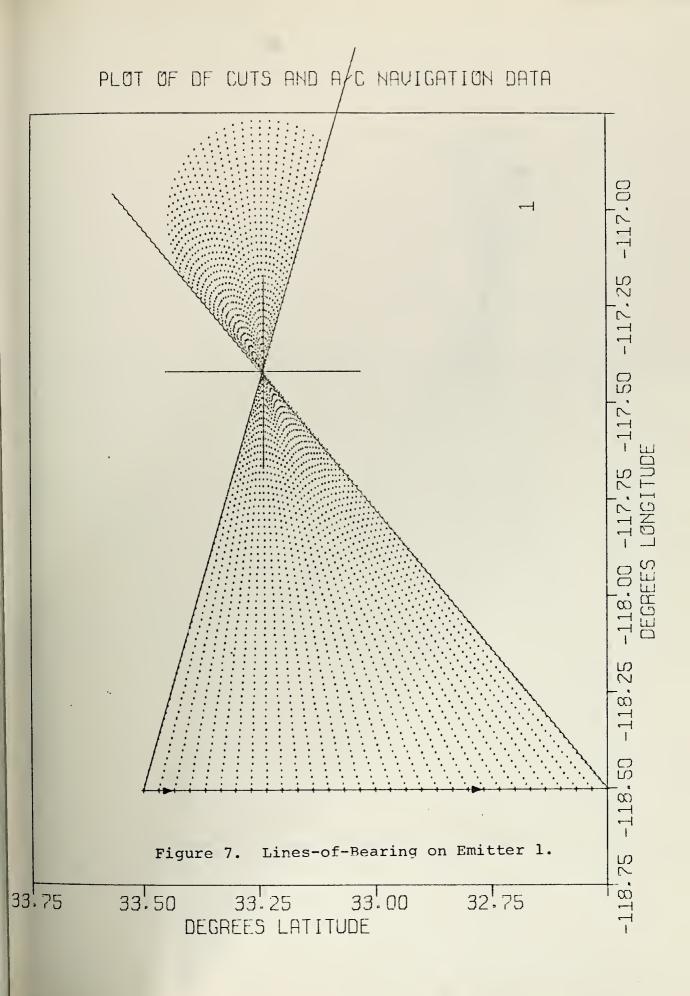
The Monte Carlo simulation developed in [2] was utilized to accomplish program validity and error analysis studies. The scenario developed for that simulation was of an aircraft flying south along the 118°W meridian at a ground speed of 600 knots and with heading of 180°. Bearing angles-of-arrival were recorded from two VORTAC stations located at

Emitter 1	Oceanside VORTAC	33.24055°N 117.41694°W
Emitter 2	San Diego VORTAC	32.78222°N 117.22444°W

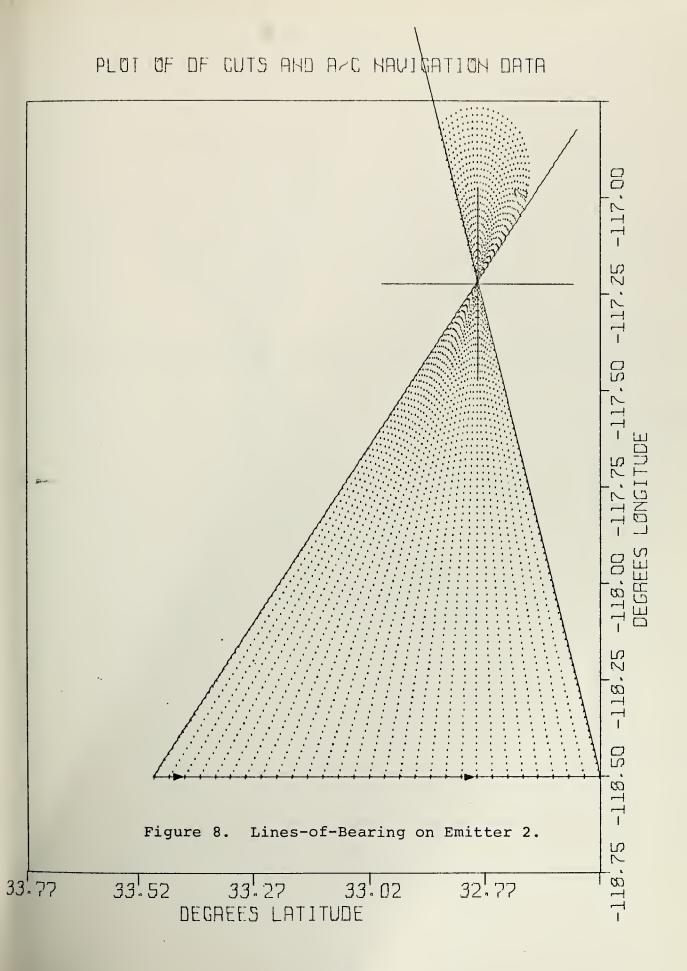
DF cuts and aircraft position fixes were recorded every six seconds alternately from each target giving a uniform sampling interval. Known true bearing angles were computed numerically from the aircraft position coordinates and the emitter position coordinates to obtain a very accurate data base. The DF cuts obtained are shown in Fig. 7 for emitter 1 and in Fig. 8 for emitter 2.

Normally distributed, zero mean random noise with an assumed variance of 1 was added independently to the latitude fixes, longitude fixes, and angles-of-arrival. The errors in aircraft heading and angle-of-arrival were combined into the single angle-of-arrival error. The flight track and noisy DF bearings are shown in Figs. 9 and 10 for emitters 1 and 2 respectively.

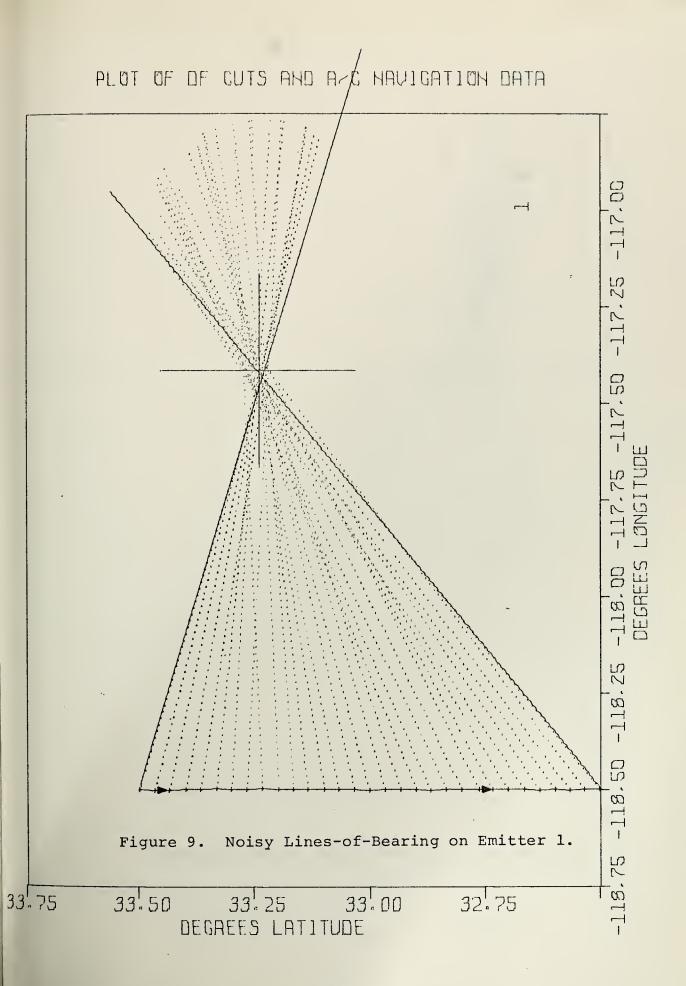




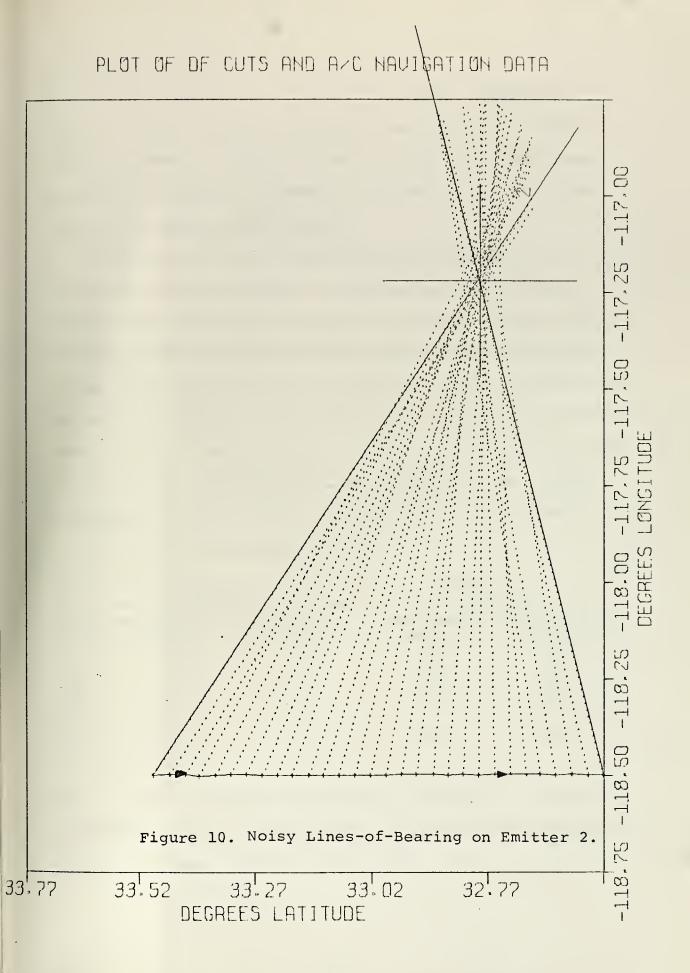














# A. ERROR ANALYSES

A large number of simulation runs were executed to obtain a statistically valid error analysis, where each run constituted a complete mission. The results of these analyses are given in Appendix E and are summarized below for the two simulation emitters.

The results of the error analysis of the navigation data filter are listed on page. The average of the noise added to the positions is shown as is the mean-square value of that noise. By comparing the listed mean-square error of the positions with that of the noise, it can be seen that the model utilized in the program has a bias which develops after a large number of runs. Each run is a complete flight of 61 position fixes, so the result after 45 runs is the statistical average of 2745 data points. The results of 2, 10, 45, and 50 runs was found to be

LATITUDE		LONGITUDE		
Runs	Mean-Square Noise	Mean-Square Error	Mean-Square Noise	Mean-Square Error
2	. 427	.326	1.80	1.21
10	.937	1.02	1.24	1.16
45	1.26	1.37	1.19	1.24
50	1.19	1.31	1.14	1.23

The units for this list are degrees times  $10^{-8}$ . The error associated with each individual position fix on a single run is shown on page .



For the analysis of the angle filter, a range error was computed from each estimated emitter position latitude error and longitude error, and all errors were statistically averaged for the total number of runs.

The longitude error which is perpendicular to the flight path was found to be much larger than the latitude error. This result agrees with the theory used to develop the ellipses which describe the initial covariance matrix for the extended Kalman filter. This result also agrees with the final orientation of the target error ellipses as shown on page 51 and 52. The results of this analysis are summarized below for 50 runs.

	AVERAGE	AVERAGE	AVERAGE
	LATITUDE	LONGITUDE	RANGE
	ERROR	ERROR	ERROR
EMITTER 1	.684	1.129	1.391
EMITTER 2	.685	1.870	2.098

The units of the above table are nautical miles.

The results of the error analysis of the extended Kalman filter are shown on page 73. The errors of each individual flight which are listed on that page point out the dependency of this filter on the accuracy of the initial estimate of the emitter position. Since the initial estimate is based on only four DF bearings for this simulation, this estimate is heavily weighted by the noise added to the first bearings and the rate of convergence of the filter. The results of the



analysis of the extended filter are given below for 50 runs with the units being nautical miles.

	AVERAGE LATITUDE ERROR	AVERAGE LONGITUDE ERROR	AVERAGE RANGE ERROR
EMITTER 1	1.535	4.654	4.988
EMITTER 2	.872	7.784	7.876

#### B. COMPUTER PROCESSING OF SIMULATED DATA

A series of plots showing the ability of this program to locate target positions in the absence of noise starts on page 50. The number in the lower right corner is the number of the DF bearing estimate associated with that target which was used to compute the target position and error ellipse in that plot. The plots on page 51, 52 and show the four points of intersection computed by POINTS which describe the error ellipse. From these plots, it can be seen that the error ellipse does accurately describe the area of the probability region associated with each position fix.

The output of a single run of the program using the simulated data starts on page 76. The first run assumes that processing will be strictly by the angle filter. The output of a run in which the extended filter was utilized starts on page 110. Plots of the error ellipses associated with the covariance matrices of each filter are included in the output.

These series of plots show the change of orientation of the error ellipse as the aircraft progressed along its flight



path. They also illustrate the decrease in area of the ellipses as more data is collected and as the angle between the smoothed initial bearing  $\hat{\theta}$  (1/N), and the filtered final bearing  $\hat{\theta}$  (N/N) increased.

By comparing the plots of the ellipse associated with the angle filter on page 50 and the plot on page 53 associated with the extended filter, it can be seen that the extended filter is initialized with the same error covariance as the angle filter.

Further comparison of the two series of plots shows that the position of the centers of the ellipses produced by the angle filter are much more erratic than those of the extended filter. This indicates that the angle filter has a faster response to each new bit of data than the extended filter. The response may seem too fast, but in order to have a rapid rate of convergence of the filter so processing could switch quickly to the extended filter, fast response is necessary, so this is a compromise.

The slow response of the extended filter is shown by the relatively slow rate of rotation of the error ellipse. The slow response is desirable to prevent the filter from converging too rapidly onto a wrong point if it is initialized with a poor initial position fix.

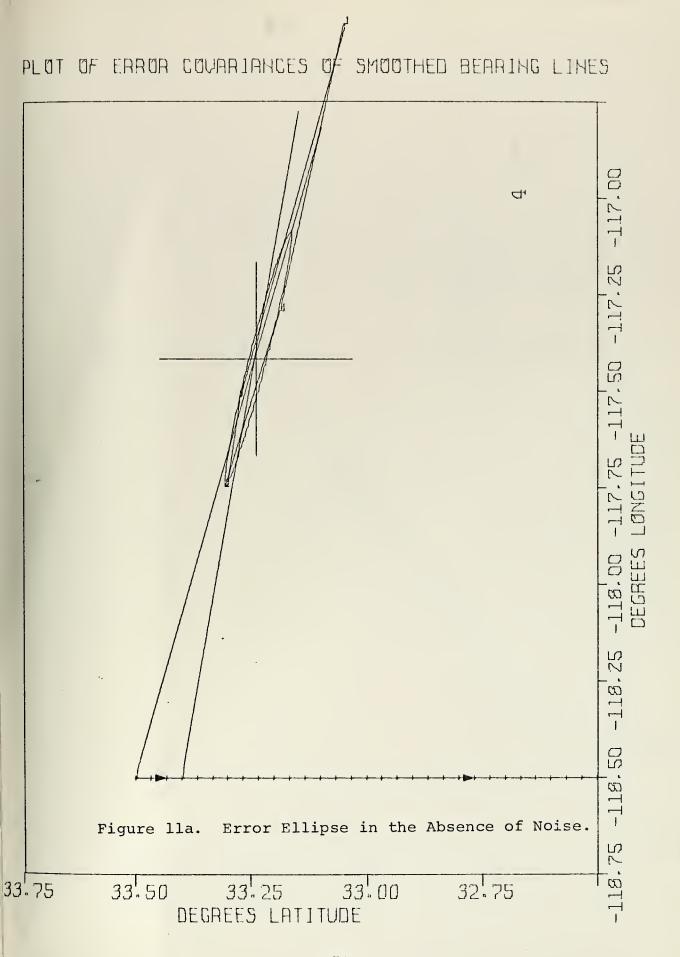
### C. AIRCRAFT FLIGHT DATA PROCESSING

The output of a computer run using data from an actual flight starts on page 144. The effectiveness of the data



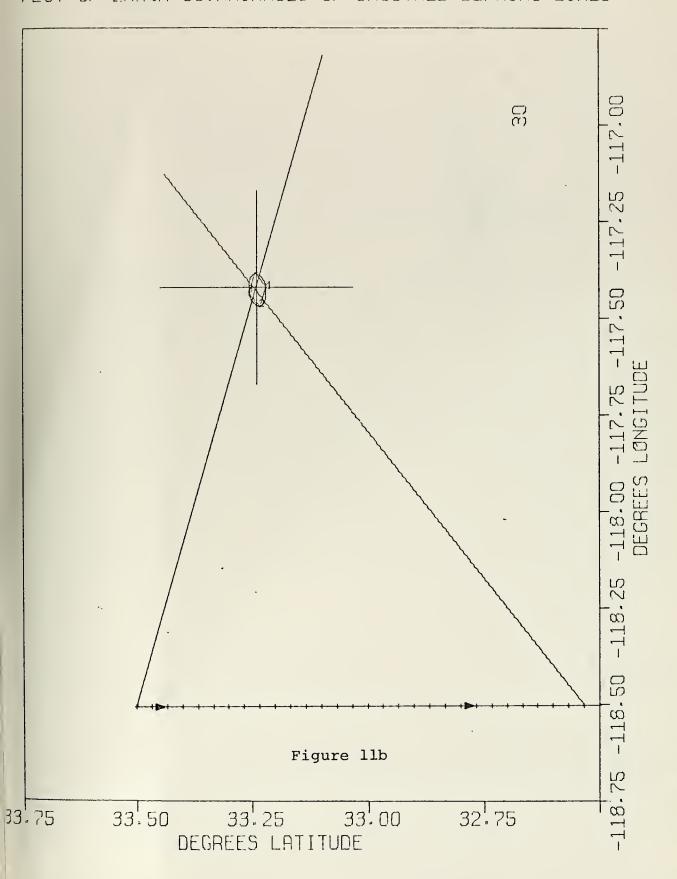
sort routine is shown by the lists on page 146 and 147. The first list is the result of the sort on PRF and FREQUENCY only. The second list is the final correlation after the data has been correlated to targets using the test discussed on page 31.





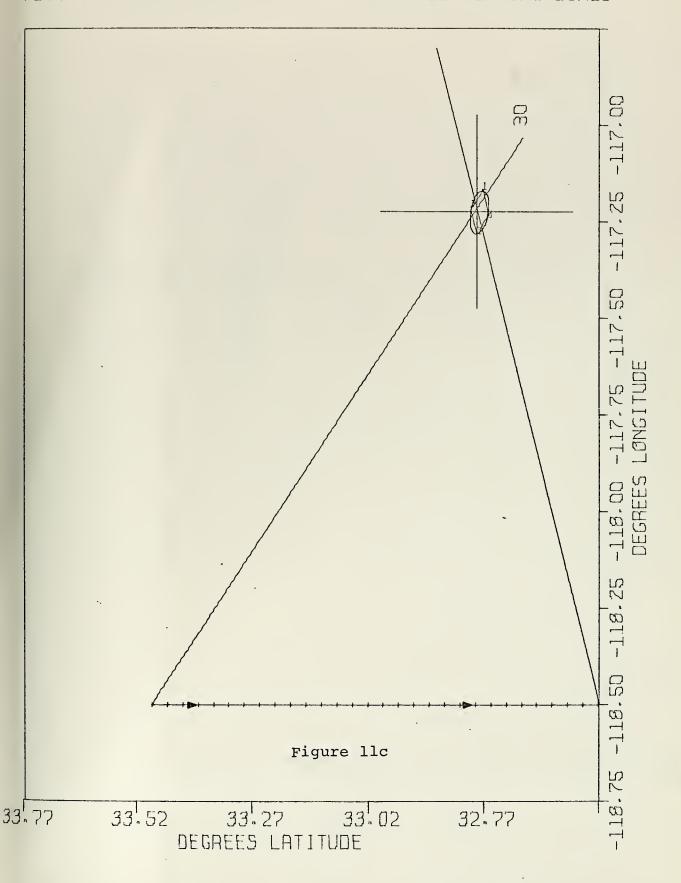


# PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES



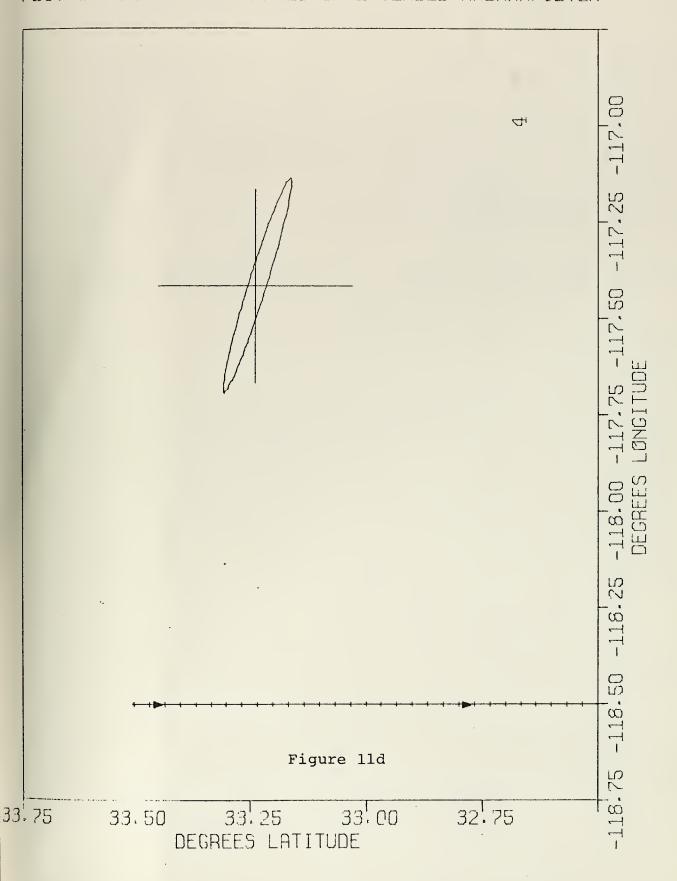


# PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES



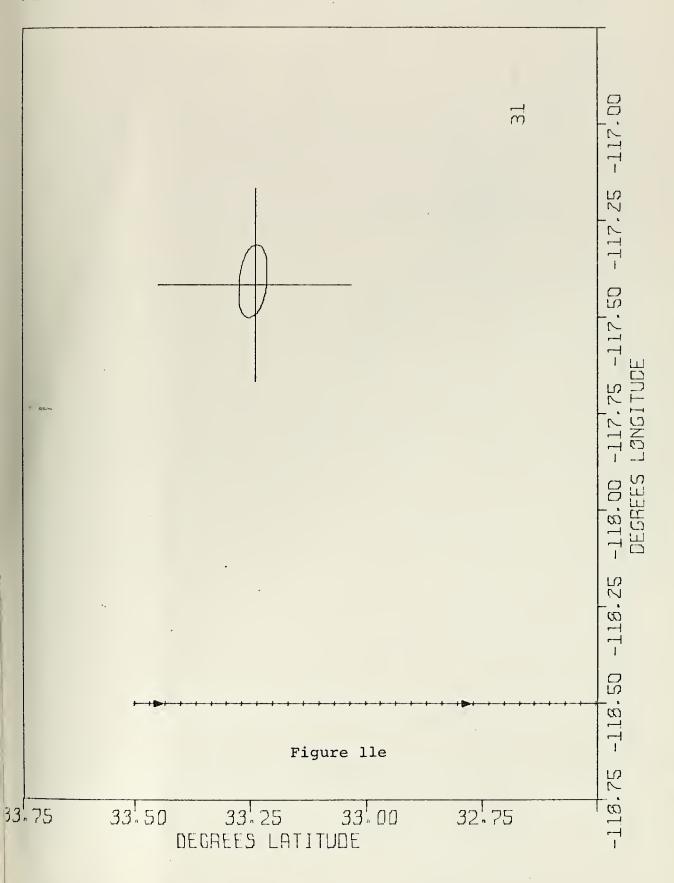


# PLOT OF ERROR COVARIANCES OF EXTENDED KALMANFILTER



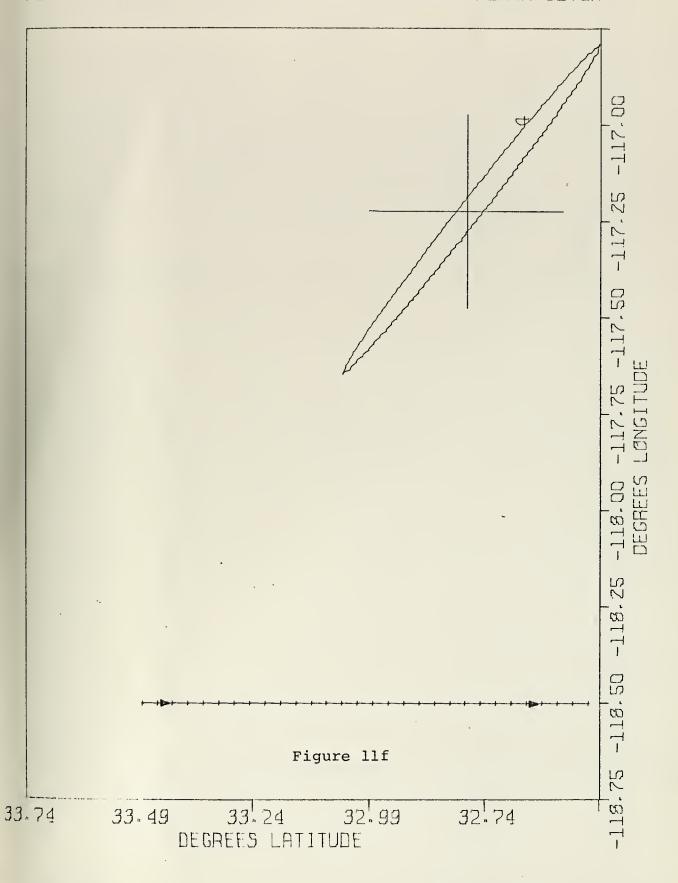


# PLOT OF ERROR COVARIANCES OF EXTENDED KALMAN FILTER



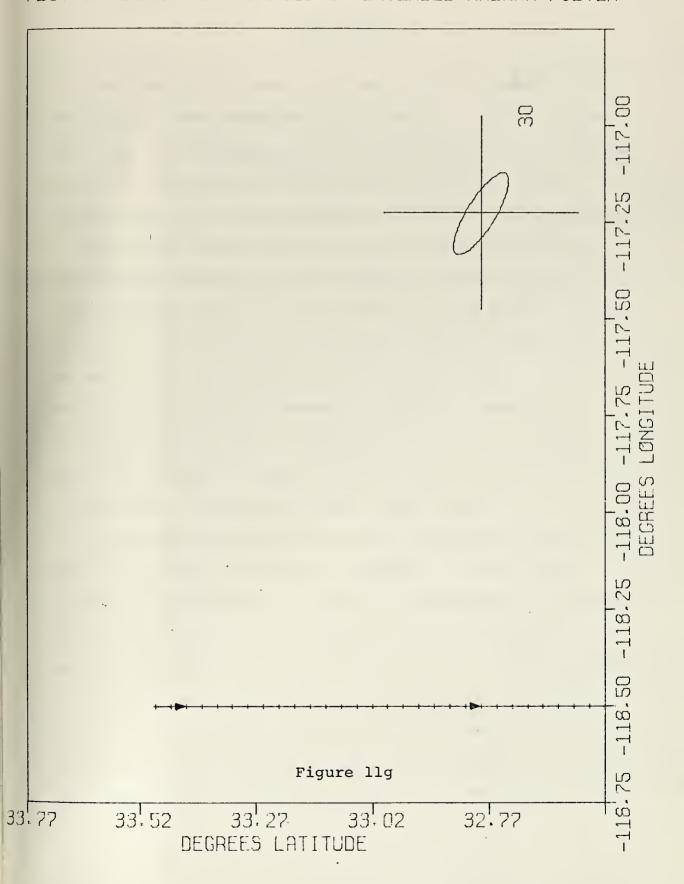


# PLOT OF ERROR COVARIANCES OF EXTENDED KALMANFILTER





# PLOT OF ERROR COVARIANCES OF EXTENDED KALMAN FILTER





#### CONCLUSIONS

The fact that Kalman filtering of digital data works is nothing new, many examples can be found in the literature.

This report has shown a particular application of the Kalman theory and presented a program which will work in that application, i.e. the processing of airborne DF information.

There are several areas of this study which need more research and testing. A more complete model for the navigation filter could be developed which would reduce the bias present in the one utilized for this report. Then it should be determined if the bias is harmful or not and if it is worth the resulting increases in computer processing to remove the small error. The response of the two emitter position locating filters should be looked at to determine if there is a more optimal Q for both schemes.

A program for plotting the error ellipses of each target for each filtering scheme was presented. This program provides a capability for displaying graphically the effect of varying the different parameters which are inputs to the subroutines, plus the effect of varying filter response time.

Many current elint systems have a digital computer as a link in their processing chain, but it is used strictly as a device to plot the data and no attempt is made to compute an optimal emitter location solution. By implementing this



Kalman filter program, the effectiveness of the elint system could be greatly enhanced by reducing manual processing time and produce more accurate target position coordinates.



#### APPENDIX A

### DERIVATION OF SCALAR KALMAN FILTER RECURSION EQUATIONS

The Kalman filter equations for all of the linear models in this program can be derived in the same general form described below. The variable names used in each individual subroutine to represent the general terms in the derivation are listed at the end of this section.

The Kalman filter recursion equations from page 10, (7) through (11), are listed below:

$$P(k|k-1) = \phi(k,k-1)P(k-1|k-1)\phi(k,k-1)^{T} + Q(k)$$
 (1A)

$$G(k) = P(k|k-1)H(k)^{T}[H(k)P(k|k-1)H(k)^{T} + R(k)]^{-1}$$
 (2A)

$$P(k|k) = P(k|k-1) - G(k)H(k)P(k|k-1)$$
 (3A)

$$\hat{X}(k|k) = \hat{X}(k|k-1) + G(k) [Z(k) - H(k)\hat{X}(k|k-1)]$$
 (4A)

$$\hat{X}(k|k-1) = \phi(k,k-1)\hat{X}(k-1|k-1)$$
(5A)

where in this application 
$$\phi(k+1,k) = \begin{bmatrix} 1 & T(k+1) \\ 0 & 1 \end{bmatrix}$$
,

$$H(k) = [1 0],$$

the P and Q matrices may be considered to have scalar components given by

$$\begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix} \text{ and } \begin{bmatrix} Q11 & Q12 \\ Q21 & Q22 \end{bmatrix} \text{ respectively,}$$

and the G matrix is a 2xl matrix of the form  $\begin{bmatrix} G1 \\ G2 \end{bmatrix}$ .



Writing (2A) in matrix notation yields

$$\begin{bmatrix}
G1 \\
G2
\end{bmatrix} = \begin{bmatrix}
P11 & P12 \\
P21 & P22
\end{bmatrix} \begin{bmatrix}
1 \\
0
\end{bmatrix} \begin{bmatrix}
1 & 0
\end{bmatrix} \begin{bmatrix}
P11 & P12 \\
P21 & P22
\end{bmatrix} \begin{bmatrix}
1 \\
0
\end{bmatrix} + R \\
; (6A)$$

and for the observation matrix  $H(k) = [1 \ 0]$ , the inverse term becomes a scalar allowing the gain terms to be computed directly as is shown below:

$$\begin{bmatrix} G1 \\ G2 \end{bmatrix} = \begin{bmatrix} P11 \\ P21 \end{bmatrix} \begin{bmatrix} P11 + R \end{bmatrix}^{-1}$$
 (7A)

which results in the scalar gain equations

G1(k) = 
$$\frac{P11(k|k-1)}{P11(k|k-1) + R(k)}$$
 (8A)

G2(k) = 
$$\frac{P21(k|k-1)}{P11(k|k-1) + R(k)}$$
 (9A)

let the P matrix be defined by

$$P(k/k-1) = E \left[\Theta(k) - \hat{\theta}(k|k-1)\right] \left[\Theta(k) - \hat{\theta}(k|k-1)\right]^{T}$$
 (10A)

from which it can be seen that since  $\widetilde{\theta}$  and  $\widetilde{\widetilde{\theta}}$  are statistically independent, where

$$\tilde{\theta} = \Theta - \hat{\theta}$$
,

then Pl2 = P21, and the P matrix is symetric. From (15) it can be seen that the Q matrix is also symetric. Equation (9A) then becomes

G2 (k) = 
$$\frac{P12 (k | k-1)}{P11 (k | k-1) + P(k)}$$
 (11A)

To solve for the prediction covariance terms, (3A) was substituted into (1A) giving



$$\begin{bmatrix}
P11 & P12 \\
P12 & P22
\end{bmatrix} = \begin{bmatrix}
1 & T \\
0 & 1
\end{bmatrix} \begin{cases}
P11 & P12 \\
P12 & P22
\end{bmatrix} - \begin{bmatrix}
G1 \\
G2
\end{bmatrix} \begin{bmatrix}
1 & 0
\end{bmatrix} \begin{bmatrix}
P11 & P12 \\
P12 & P22
\end{bmatrix} \begin{bmatrix}
1 & 0
\end{bmatrix} + \begin{bmatrix}
Q11 & Q12 \\
Q12 & Q22
\end{bmatrix} \\
k \mid k - 1$$
(12A)

# VARIABLE NAMES USED IN PROGRAM TO REPRESENT VARIABLES IN GENERAL DERIVATION

General Variable	R	P	P	G	Х	X	Х	E	${f z}$
Subroutine Name									
NAV LATITUDE LONGITUDE	RNAV	SP	SPKK	SG	SLAD SLOD	VELED VELND	SLATD SLOND	ELAT ELON	ACLAD ACLOD
GEORGE	RCUT	P	-	G	THTD	TDTD	TPTD	E	THETAD
EXTEND LATITUDE LONGITUDE LINEARIZEI	EXTEND	EP	-	G	XTD YTD	XTDDOT YTDDOT	XTDHAT YTDHAT	EX EY	XTD1 YTD1
MEASURE-		P	-		XTD1 YTDL	-	TX TX	ER ER	THETA THETA



#### APPENDIX B

#### DERIVATION OF SMOOTHING FILTER EQUATIONS

#### A. FIXED POINT SMOOTHING

The equations used to smooth the estimates of the first DF bearing, THTD1, for a given target are listed on page 15 and are repeated below with k=1.

### 1. Filter Equation

$$\hat{x}(1|j) = \hat{x}(1|j-1) + W(j)H(j)^{T}R^{-1}(j) \left[z(j) - H(j)\phi(j,j-1)x(j-1|j-1)\right]$$
(1B)

where j = 2, 3, ..., and the initial condition is  $\hat{X}(1,1)$ .

### 2. Gain Equation

$$W(j) = W(j-1)\phi(j,j-1)^{T} \left[I-S(j)P(j|j)\right]$$
 (2B)

where W(1) = P(1|1) and  $S(j) = H(j)^{T}R^{-1}(j)H(j)$ 

### 3. Covariance Equation

$$P(1|j) = P(1|j-1) - W(j) \left[S(j)P(j|j-1)S(j) + S(j)\right] W(j)^{T}$$
(3B)

where the initial condition is P(1|1). P(j|j) and P(j|j-1) are the error covariance matrices from the optimal filter and predictor respectively. Since  $H = [1 \ 0]$  and R(j) is a scalar, these matrix equations become scalar expressions.

To compute a smoothed estimate of  $\hat{X}(1|j)$ , it is first necessary to solve (2B) for the values of the gains to be used in (1B).



First solve for S(j)

$$S(j) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{1}{R} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1/R & 0 \\ 0 & 0 \end{bmatrix}$$

$$(4B)$$

Then noting that

$$\phi(j,j-1) = \begin{bmatrix} 1 & 0 \\ T(j) & 1 \end{bmatrix}$$
 (5B)

Equation (2B) becomes

$$\begin{bmatrix} W11 & W12 \\ W21 & W22 \end{bmatrix} = \begin{bmatrix} W11 & W12 \\ W21 & W22 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ T & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1/R & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P11 & P12 \\ P12 & P22 \end{bmatrix}$$
(6B)
$$(1/j) \qquad (1/j-1) \qquad (j/j)$$

To solve (1B) first note that

$$\hat{\mathbf{z}}(\mathbf{j}) = \mathbf{H}(\mathbf{j}) \phi \hat{\mathbf{x}}(\mathbf{j} - \mathbf{1} | \mathbf{j} - \mathbf{1}) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \mathbf{x}_1 + \mathbf{x}_2 \mathbf{T} \quad (7B)$$

so therefore let  $E(j)=Z(j)-X_1(j)-X_2(j)T(j)$ . Then in the notation used in the program

$$\begin{bmatrix}
THTD1 \\
TDTD1
\end{bmatrix} = \begin{bmatrix}
THTD1 \\
THTD1
\end{bmatrix} + \begin{bmatrix}
W11 & W12 \\
W21 & W22
\end{bmatrix} \begin{bmatrix}
1 \\
0
\end{bmatrix} \frac{E(j)}{R}.$$
(8B)

The error covariance matrix for the fixed point smoothing filter is

$$D = \begin{bmatrix} D11 & D12 \\ D21 & D22 \end{bmatrix}$$
 (9B)

so (3B) written matrix form is



$$\begin{bmatrix}
D11 & D12 \\
D21 & D22
\end{bmatrix} = \begin{bmatrix}
D11 & D12 \\
D21 & D22
\end{bmatrix} - \begin{bmatrix}
W11 & W12 \\
W21 & W22
\end{bmatrix} \begin{bmatrix}
1/R & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
P11 & P12 \\
P12 & P22
\end{bmatrix} \\
(1|j) & (1|j-1)$$

$$\begin{bmatrix}
1/R & 0 \\
0 & 0
\end{bmatrix} + \begin{bmatrix}
1/R & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
W11 & W21 \\
W12 & W22
\end{bmatrix} (10B)$$

#### B. FIXED INTERVAL SMOOTHING

The fixed interval smoothing equations are listed on page and are repeated below.

### 1. Filter Equation

$$\hat{X}(k|n) = \hat{X}(k|k) + A(k) \left[\hat{X}(k+1|n) - \hat{X}(k+1|k)\right]$$
 (11B)

for k = n-1, n-2, ..., 0, where  $\hat{X}(n|n)$  is the boundary condition for k = n-1.

### 2. Gain Equation

$$A(k) = P(k|k) \phi (k+1,k)^{T} P^{-1}(k+1|k)$$
 (12B)

## 3. Covariance Equation

$$P(k|n) = P(k|k) + A(k) \left[ P(k+1|n) - P(k+1|k) \right] A(k)^{T}$$
 (13B)

The covariance equation is not used in the computation of the smoothed estimate of  $\hat{X}(k|n)$  or filter gain A(k), so it was not utilized in the program.

First the values of the gain matrix A, are computed. Using the notation  $PIN=P^{-1}$  (12B) becomes

$$\begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix} = \begin{bmatrix} P11 & P12 \\ P12 & P22 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ T & 1 \end{bmatrix} \begin{bmatrix} PIN11 & PIN12 \\ PIN12 & PIN22 \end{bmatrix}$$
(14B)

where P(k|k) and P(k+1|k) are the covariance terms from the optimal filter equations and the fact that P12=P21 is utilized.



Again letting  $E(k) = \hat{X}(k+1|n) - H(k)\phi(k+1|k)\hat{X}(k|k)$ , (11B) becomes

$$\hat{X}(k|n) = \hat{X}(k|k) + A(k)E(k)$$
 (15B)

and (11B) is written in the notation used in the program as

$$\begin{bmatrix} SLAPSM \\ VELNDSM \end{bmatrix} = \begin{bmatrix} SLAD \\ VELND \end{bmatrix} + \begin{bmatrix} A11 & A12 \\ A21 & A22 \end{bmatrix} \begin{bmatrix} ELAD1 \end{bmatrix}$$
 (16B)

and

$$\begin{bmatrix} \text{SLODSM} \\ \text{VELEDS} \end{bmatrix} = \begin{bmatrix} \text{SLOD} \\ \text{VELED} \end{bmatrix} + \begin{bmatrix} \text{All} & \text{Al2} \\ \text{A21} & \text{A22} \end{bmatrix} \begin{bmatrix} \text{ELON1} \end{bmatrix}. \tag{17B}$$



#### APPENDIX C

# DERIVATION OF THE VECTOR METHOD POSITION LOCATING ALGORITHM

This algorithm is divided into three basic sections;

(A) computation of normal vectors to the bearing plane, (B) computation of bearing vectors, and (C) computation of the target position vector.

# A. COMPUTATION OF THE NORMAL VECTOR TO THE BEARING PLANE

The normal vector of the bearing planes are computed as described in Section II,D, using the aircraft latitude and longitude  $(\phi,\theta)$ , and the DF bearing angle  $\alpha$ .

The bearing plane has the equation dxx + dyy + dzz = 0. The normal vector, N, can be written in the form

$$N = dxi + dyj + dzk$$
 (1C)

where i,j,k are the unit base vectors in the x,y,z directions. The components dx, dy, and dz, of normal vector N correspond to the coefficients found from (27). That is

$$dx = -\sin\alpha \sin\theta - \cos\alpha \cos\theta \sin\phi \qquad (2C)$$

$$dy = \sin\alpha \cos\theta - \cos\alpha \sin\theta \sin\phi \tag{3C}$$

$$dz = \cos \alpha \cos \phi \tag{4C}$$

#### B. COMPUTATION OF BEARING VECTOR

Let SI(x,y,z) = xI + yj + zk denote the aircraft position vector. The cross product of SI and N produce the vector



perpendicular to the plane of both SI and N which is the bearing vector D(I), thus

$$D(I) - SI \times N = \begin{vmatrix} 1 & j & k \\ x & y & z \\ dx & dy & dz \end{vmatrix}$$
 (5C)

so that D(I) = A(I)i + B(I)j + C(I)k, I=1,2, where

$$A = y \times DZ - z \times DY \tag{6C}$$

$$B = z \times DX - x \times DZ \tag{7C}$$

$$C = x \times DY - y \times DX. \tag{8C}$$

Then each of the vectors is normalized to determine the length of the unit vector by dividing by D where

$$D = \sqrt{A^2 + B^2 + C^2} . {(9C)}$$

#### C. COMPUTATION OF THE TARGET POSITION VECTOR

The two bearing vectors D(1) and D(2) are cross multiplied to find the x,y,z coordinates of the target vector. So

$$X = D(1) \times D(2) = \begin{vmatrix} x & y & z \\ A1 & B1 & C1 \\ A2 & B2 & C2 \end{vmatrix}$$
 (10C)

The algorithm for finding the target position vector is therefore

$$X1 = B1 \times C2 - B2 \times C1$$
 (11C)

$$X2 = A2 \times C1 - A1 \times C2$$
 (12C)

$$X3 = A1 \times B2 - A2 \times B1$$
 (13C)



This vector is normalized to a unit vector with one unit being the radius of the earth.

To find the latitude and longitude coordinates of the target position the following equations are used

TLAD = 
$$\tan^{-1}\left(\frac{x_3}{\sqrt{x_1^2 + x_2^2}}\right)$$
 (14)

$$TLOD = tan^{-1} \left( \frac{X2}{X1} \right)$$
 (15)

### D. EXAMPLE OF VECTOR METHOD FOR LOCATING TARGET POSITION

A DF bearing of 30 degrees is taken when the aircraft is at latitude 30 degrees north and longitude 45 degrees east.

A second DF bearing of 60 degrees is taken from the same emitter when the aircraft is at 31 degrees north and 45 east.

l. First find the normal vector to the bearing plane by substituting the values of  $\phi$  ,  $\theta$  , and  $\alpha$  into (2C) through (4C).

DX = 
$$-\sin 30 \sin 45 - \cos 30 \cos 45 \sin 30 = -.6597$$
  
DY =  $\sin 30 \cos 45 - \cos 30 \sin 45 \sin 30 = .0473$   
DZ =  $\cos 30 \cos 30 = .750$ 

2. Using (21) through (23) find the x,y, and z components of the aircraft position vector.

$$x = \cos 30 \cos 45 = .6123$$
  
 $y = \cos 30 \sin 45 = .6123$   
 $z = \sin 30 = .500$ 



3. Next find the bearing vector, D(I), which is the cross product of the aircraft position vector and the normal to the bearing plane, using (6C) through (8C).

Al = 
$$.6123 \times .750 - .50 \times .0473 = .4356$$
  
Bl =  $.50 \times (-.6597) - .6123 \times .750 = -.7872$   
Cl =  $.6123 \times .0473 - .6123 \times (-.6597) = .4329$ 

4. Find the value of D that will normalize D(I) to a unit vector. Using (9C)

$$D = \sqrt{.1897 + .6228 + .187} = 1.0$$

Therefore, the bearing vector is already a unit vector.

- 5. Similarly the corresponding values for the second bearing are: DX = .7944, DY = .4302, DZ = .42855, x = .6061, y = .6061, and z = .5150.
- 6. Since the angle of arrival of the second DF bearing is larger than the angle of the first bearing, the second bearing must be crossed into the first to obtain the outward pointing position vector. Therefore from (9C) and (11C) through (13C)

$$X1 = (-.6688) \times .4329 - (-.7892) \times .7422 = .2962$$
  
 $X2 = .4356 \times .7422 - .0381 \times .4329 = .3068$   
 $X3 = .0381 \times (-.7892) - .4356 \times (-.6031) = .2613$   
 $D = \sqrt{.0877 + .0941 + .0682} = .50$ 

7. Find the longitude and latitude coordinates of the target position using (14C) and (15C).



TLOD = ARCTAN(.6136/.5924) = 46.0 degrees TLAD = ARCTAN(.5224/.8529) = 31.5 degrees

The computer solution to this problem was TLOD = 46.01540 and TLAD = 31.49590.



#### APPENDIX D

#### DERIVATION OF EXTENDED KALMAN FILTER EQUATIONS

#### A. ERROR COVARIANCE EQUATION INITIALIZATION

The ellipse described by the Pll and Dll terms in the angle filtering routine lies in the X'Y' coordinate system which is rotated counter-clockwise (90 -  $\alpha$ ) degrees from the XY coordinate system used in the Extended Kalman Filter. To determine the values of the axes of the ellipse in the XY coordinate system, the transformation T, described by (51) was utilized.

$$T = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$
 (1D)

Let the semi-major axis be  $A' = E[X' - \hat{X}']$ , and the semi-major axis be  $C' = E[Y' - \hat{Y}']$ . Define Z = TX where  $X^T = [A' \ C']$ , then the error covariance matrix P, in the XY frame is

$$P = E\left[Z \ Z^{T}\right] = T E\left[X \ X^{T}\right]T^{T}. \tag{2D}$$

Writing this equation in matrix form yields

$$\begin{bmatrix}
P11 & P12 \\
P12 & P21
\end{bmatrix} = \begin{bmatrix}
\cos\alpha & -\sin\alpha \\
\sin\alpha & \cos\alpha
\end{bmatrix} \begin{bmatrix}
A' \\
C'
\end{bmatrix} \begin{bmatrix}
A' & C'
\end{bmatrix} \begin{bmatrix}
\cos\alpha & \sin\alpha \\
-\sin\alpha & \cos\alpha
\end{bmatrix} (3D)$$

Carrying out the indicated matrix multiplications in (3D) produces (52) through (54) which are repeated here.

$$P11(KI) = (A \sin\alpha)^{2} + (C \cos\alpha)^{2}$$
 (4D)



$$P12(KI) = \sin\alpha \cos\alpha (A^2 - C^2)$$
 (5D)

$$P22(KI) = (A \cos\alpha)^{2} + (C \sin\alpha)^{2}$$
 (6D)

#### B. LATITUDE AND LONGITUDE FILTER EQUATIONS

The block diagram for the filter system used in subroutine Extend is shown in Fig. 1. As stated in Section III, part F, the latitude and longitude filters are identical to the corresponding filters used in Subroutine Nav to compute the optimal aircraft position estimates. The recursion equations for these filters are derived in Appendix A with the variable names utilized in this subroutine being listed at the end of Appendix A.

### C. LINEARIZED MEASUREMENT EQUATIONS

The extended Kalman filter recursion equations (36) through (40) are similar in form to the basic Kalman filter recursion equations on page , except that the observation matrix is a nonlinear transformation matrix M as defined by (35) where

$$M(k) = \frac{\partial \hat{\theta}}{\partial \hat{X}} \Big|_{\hat{X}} = \left[ \frac{\partial \hat{\theta}}{\partial \lambda_{T}} \quad \frac{\partial \hat{\theta}}{\partial L_{T}} \right] = \left[ DMX \quad DMY \right]$$
 (7D)

Taking partial derivatives of (31) yields

$$DMX = \frac{\partial \hat{\theta}}{\partial \lambda_{T}} = \frac{(L_{T}-L) \cos L_{T}}{(L_{T}-L)^{2} + (\lambda_{T}-\lambda)^{2} \cos^{2}L_{T}}$$
(8D)

$$DMY = \frac{\partial \hat{\theta}}{\partial L_{T}} = \frac{-(\lambda_{T} - \lambda) \left[ (L_{T} - L) \sin L_{T} + \cos L_{T} \right]}{(L_{T} - L)^{2} + (\lambda_{T} - \lambda)^{2} \cos^{2} L_{T}}$$
(9D)



These terms are computed numerically and substituted into equations (36) and (37) which are rewritten in matrix form as

$$\begin{bmatrix}
GX \\
GY
\end{bmatrix} = \begin{bmatrix}
P11 & P12 \\
P12 & P22
\end{bmatrix} \begin{bmatrix}
DMX \\
DMY
\end{bmatrix} \begin{bmatrix}
DMX & DMY
\end{bmatrix} \begin{bmatrix}
P11 & P12 \\
P12 & P22
\end{bmatrix} \begin{bmatrix}
DMX \\
DMY
\end{bmatrix} + R$$
(10D)

and

$$\begin{bmatrix}
P11 & P12 \\
P12 & P22
\end{bmatrix} = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} \begin{bmatrix}
P11 & P12 \\
P12 & P22
\end{bmatrix} \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix} + \begin{bmatrix}
Q11 & Q12 \\
Q12 & Q22
\end{bmatrix}$$

$$k + 1 \mid k \qquad k \mid k$$
(11D)

Equation (38) is then substituted into (10D); however, the fact that the  $\phi$  matrix is the identity matrix considerably simplifies the recursion equations since they are no longer dependent on the sampling interval T(k). Since the gain terms are computed as a function of the nonlinear terms DMX and DMY, it is essential to have the correct units for the error term ER(k). In this case the units of gain are not dimensionless but degrees per radian, so the error term must be given in radians to update the target position estimates in degrees of latitude and longitude.

Equation (40) is substituted into (39) and written in the notation used in the program is

$$XTDl(k) = XTD(k) + GX(k)ER(k)$$
 (12D)

and

$$YTD1(k) = YTD(k) + GY(k)ER(k).$$
 (13D)



## APPENDIX E

## MONTE CARLO SIMULATION ERROR ANALYSIS

## ESTIMATED AIRCRAFT POSITION ERROR IN DEGREES LATITUDE

	LATI	TUDE	LON	GITUDE
RUN #	NOISE ADDED TO POSITION	SMOOTHED POSITION ERROR	NOISE ADDED TO POSITION	SMOOTHED POSITION ERROR
1234567890123444444444444444444444444444444444444	-0.921E-04 0.775E-04 0.7748E-04 -0.483E-04 -0.483E-04 -0.885E-04 -0.138E-03 -0.138E-03 -0.138E-03 -0.138E-03 -0.138E-04 -0.138E-04 -0.138E-04 -0.138E-04 -0.1583E-04 -0.17588E-04 -0.17588E-04 -0.17588E-04 -0.17588E-04 -0.17588E-04 -0.1888E-03 -0.1948E-03 -0.1948E-03 -0.1948E-03 -0.1958E-04 -0.1888E-03 -0.1958E-04 -0.1888E-03 -0.1958E-04 -0.1888E-03 -0.1958E-04	-0.708E-04 0.388E-04 0.958E-04 0.958E-04 0.114E-03 -0.548E-04 0.528E-04 0.528E-04 0.128E-03 -0.8825E-04 0.128E-03 -0.128E-03 -0.128E-03 -0.128E-04 0.125E-04 -0.125E-04 -0.125E-04 -0.125E-04 -0.125E-03	0.135E-033 -0.378EE-033 -0.378EE-0044 -0.378EE-0044 -0.158EE-0044 -0.1555EE-0044 -0.15555EE-0044 -0.1505EE-0044 -0.1505EE-0044 -0.1505EE-0044 -0.1505EE-0044 -0.1505EE-0044 -0.1505EE-0044 -0.1006 -0.	0.1175=-03 -0.613E04 -0.182E04 -0.182E04 -0.182E04 -0.182E04 -0.182E04 -0.182E04 -0.182E05 -0.182E06 -0.182E07 -0.182E
		AV ER AGE	ERRORS	
ATITU	NO I SE JDE	MEAN SQUARE NOISE	ERROR	MEAN SQUARE ERROR
ONGIT	0.814E-05 TUDE	0.126E-07	0.325E-04	0.137E-07
	0.552E-05	0.119E-07	-0.205E-04	0.124E-07



# MONTE CARLO SIMULATION ERROR ANALYSIS ESTIMATED EMITTER POSITION ERROR IN NAUTICAL MILES EXTENDED KALMAN FILTER

RUN	DL AT 1	EMITTER #	1 DRNG1	DLAT2	EMITTER # DLON2	2 DRNG2
1234567890123456789012345678901234567890	-0.13275 -0.141333333447344734473 -0.44853333458679621513348896760.238698194730.0137360.2386961962570.01518889760.1518889760.1518888976412194314770.1518489910.1518489910.1518489910.1518489910.151849910.1518488970.1518488970.1518488970.1518488970.1518488970.1518488970.1518488970.1518488970.1518488970.1518488970.15184888970.15184888970.1518489970.151848970.1518489970.1518489970.1518489970.1518489970.1518489970.1518489970.151849970.151849970.151849970.151849970.151849970.151849970.151849970.151849970.1518499970.1518499970.1518499970.1518499970.1518499970.1518499970.1518499999999999999999999999999999999999	75-24-15-15-13-26-3-1-15-13-13-14-12-1-15-13-13-14-12-1-15-13-13-13-13-13-13-13-13-13-13-13-13-13-	4484418441850688579988571379175868879598857137917586879598857137917586879598857959885713879599999999999999999999999999999999999	0.07599 0.19043 0.19043 -0.10895	-4.61521 4.615221 4.615221 17.5221 17.5221 17.5221 17.5221 17.5221 17.5221 17.5221 17.5221 17.5221 17.5221 17.5222 17.5237	4.617177 4.66177177 4.66177177 7.66177177 7.66177177 7.66177177 7.66177177 6.1981044 10.9981044 10.998104 10.998104 10.998104 10.99812109961 10.99812109961 10.99812109961 10.9987251 10.998121463561 10.998121463561 10.998121463561 10.998121463561 10.998121463561 10.998121463561 10.998121463561 10.998121463561 10.998121463561 10.998121463561 10.9981214635555 10.9981214635555 10.9981214635555 10.9981214635555 10.9981214635555 10.9981214635555 10.9981214635555 10.9981214635555 10.9981214635555 10.99812146355555 10.9981214635555 10.9981214635555 10.998121463561 10.99812146361 10.99812146361 10.99812146361 10.99812146361 10.99812146361 10.99812146361 10.99812146361 10.99812146361 10.998121461 10.998121461 10.998121461 10.998121461 10.998121461 10.998121461 10.998121461 10.998121461 10.998121461 10.998121461 10.
	AVERAGE		-KRUR IN N AVERAGE	AUTICAL MI AVERAGE		AVERAGE
RUN	LATITUD	E LONGITUD ERROR		LATITUE ERROR	DE LONGITUDE ERROR	E RANGE ERROR
50	1.53497	4.65418	4.98781	0.87208	7.78420	7.87609



# MONTE CARLO SIMULATION ERROR ANALYSIS ESTIMATED EMITTER POSITION ERROR IN NAUTICAL MILES BEARING ANGLE-OF-ARRIVAL FILTER

RUN	DL AT 1	EMITTER #	1 DRNG1	DLAT2	EMITTER # 2 DLON2	DRNG2
1234567890123456789012345678901234567890123456789	-0.77362 -0.81299 -0.70770 1.20026 0.11078 -0.215635 0.227466 0.19409 -0.811105 0.410409 -0.8111165 0.1474565 -1.28174 -0.54565 -0.04211 1.28174 -0.70129 0.74799 -0.74799 -0.917811 -0.917811 -0.917811 -0.917811 -0.917811 -0.917811 -0.917811 -0.917811 -0.917811 -0.917811 -0.917811 -0.9178331 -0.9178331 -0.9178331 -0.917831 -0.91	-0.96022 -0.41349 -1.31369 -1.34248 -1.3546866 -1.718995 -0.164259 -1.55573905 -1.46250336 -1.55573905 -1.46250336 -2.44251333 -2.44251333 -2.44251333 -2.345333 -2.345333 -2.345333 -2.345333 -2.345333 -2.345333 -2.345333 -1.155516944 -1.15516944 -1.17237492 -1.1891625	1.910404065 3010 1.49104065 1.49104065 1.49104065 1.49104065 1.49104065 1.49104065 1.49104065 1.49104065 1.49104065 1.49104065 1.4910406 1.49104	-0.180333 -0.603336 -0.603336 -0.624783 -0.624783 -0.8075992 -0.8075992 -0.180422 0.6669444 -0.2066654 -0.37829072 -0.6792170 -0.37829072 -0.394699 -1.617272 -0.9389773 -0.9389772 -0.94551237 -0.94551237 -0.94551237 -0.6664441 -0.9755936 -0.6664441 -0.9755936 -0.6652140 -0.6652140 -0.66551249 -0.66551249 -0.66551249 -0.66551249 -0.66551249 -0.6655144 -0.9453945 -0.945394 -0.945394 -0.945394 -0.945394 -0.945394 -0.945394 -0.945394 -0.945394 -0.945394 -0.945394 -0.945394 -0.9	-0.61364569 1.61364569 2.643647369336998 2.6494376953370558 2.6494376953370558 2.6232344776959333910 2.6232394365467303346673036673 2.6244774692885166744499 2.626467447499286516938639448677256444499 2.6264678725564678 2.6264678725564678 2.626467872556474992885169838839968877256888516988859783888399688772568885978388399688772568885978388839968877256888597838883996887725688859783888859783888859783888898888888888	0.18407 1.1944679 1.1944107 1.57115707510 1.5711799457 2.1811337223207 1.636923207 1.636923207 1.636923207 1.636923207 1.6369236691 1.6369236691 1.636923691 1.636923691 1.636923691 1.636923691 1.636923691 1.636923691 1.63692333333660 1.636923691 1.636923333333660 1.636923691 1.6369
RUNS	AVERAGE LATITUDE ERROR	AVERAGE E LONGITUDE ERROR	AVERAGE RANGE ERROR	AVERAGE LATITUD ERROR	AVERAGE E LONGITUDE ERROR	AVERAGE RANGE ERROR
50	0.68492	1.12906	1.39129	0.68879	1.87039	2.09753



## LISTING OF EMITTER TARGET DATA

K TIMET FREO PR	= PW	BRNGD	THETAD
1 100.0 1197.0 1500 2 106.0 1212.0 2500 4 118.0 1212.0 2500 6 130.0 1212.0 2500 8 142.0 1212.0 2500 10 154.0 1212.0 2500 11 166.0 1212.0 2500 11 166.0 1212.0 2500 11 172.0 1500 11 166.0 1212.0 2500 11 172.0 1500 11 160.0 1197.0 1500 11 172.0 1500 12 166.0 1212.0 2500 13 172.0 1212.0 2500 14 178.0 1212.0 2500 15 184.0 1197.0 1500 16 190.0 1212.0 2500 17 196.0 1197.0 1500 18 202.0 1212.0 2500 21 220.0 1197.0 1500 22 226.0 1212.0 2500 23 232.0 1197.0 1500 24 238.0 1212.0 2500 25 244.0 1197.0 1500 26 250.0 1212.0 2500 27 256.0 1197.0 1500 30 274.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2500 31 280.0 1212.0 2550 31 298.0 1212.0 2550	00000000000000000000000000000000000000	105.4466932154486932164466932111020.6687737310421215488405787311112122.688405787311112122.688405787311112121.6999.6987399133999.6999.6999.6999.6999.6999.6999.	14666293361155533679155221273667913772244922214212121212121212121212121212121



## AIRCRAFT NAVIGATION COMPUTER DATA

	LATITUDE	NOI SY LATITUDE	SMOOTHED LATITUDE	ACTUAL LONGITUDE	NOISY LONGITUDE	SMOOTHED LONGITUDE
1234567890123456789012345678901234567890123456789012345678901	0037037037037037037037037037037037037037	719527967826544075713482222009281515381898882724176103070590447036936926937922222200928151538189988827241761030705904703369369269379269369268350189480359388227026947004693695444444443333333333333333333333333	711644361465829521286357244789379360111756110428988622825555604904670366926936936936936936936936936936936936936936	-118.50000 -118.50000 -118.50000 -118.50000 -118.50000 -118.50000 -118.50000 -118.50000 -118.50000 -118.5500000 -118.5500000	-118.500957 -118.5009972666447099972-118.50099999999999999999999999999999999999	-118. • 550000000000000000000000000000000000



## NAVIGATION DATA FILTER PARAMETERS

K	SG1	SG2	SP11	SP12	SP22	Т
12345678901234567890123456789012345678901234567890123456789012345678901	0.99509531666666666666666666666666666666666666	0.0 0.16216 0.16246 0.11174 0.10864 0.10906 0.10901	1.00000 37.00000 37.00000 0.97585 0.003337 0.00261	0.0 0.124333333333333333333333333333333333333	1.00004 0.00004 0.00004 0.00009 0.00008	00000000000000000000000000000000000000



## NAVIGATION DATA FILTER PARAMETERS

K	VELND	ELAT	SLATD	VELED	ELON	SLOND
5678901234567890123456789012333333344443	-0.00285	0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0	88804468890148877293886236999146253669515388333333333333333333333333333333333	0.00003 0.00001 0.000012 0.000012 0.000012 0.000014 0.000014 0.000014 0.000005 0.000014 0.0000014 0.0000014 0.0000014 0.0000014 0.0000014 0.0000014 0.0000014 0.0000014 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000015 0.0000001 0.0000000000000000000000000	0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0	-118.5009984 -118.50001921 -118.50001921 -118.50001921 -118.50001921 -118.50003973297 -118.50003973297 -118.5000034 -118.5000034 -118.5000034 -118.5000034 -118.5000034 -118.5000991 -118.5000991 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000990 -118.5000



1	J	К	P11(K)	P12(K)	G1(K)	G2(K)
111111111111111111111111111111111222222	02345678901234567890123456789010234567890112345678901234567890 11111111122222222222233	13579135791357913579135791357912468024680246802468024680 1111122222233333334444455555556 1111122222233333334444455555556	10000.0000 1440001.00000 6.8125 2.7990 1.9142 1.4289 1.1400 0.96348 0.7893 0.7517 0.7121 0.7164 0.7163 0.7164 0.71659 0.7157 0.7157 0.7157 0.7157 0.7157 10000.0000 6.8125 2.7990 1.9142 1.4289 1.1400 144001.0000 6.8125 2.7990 1.9142 1.4289 1.161 0.7164 0.8548 0.8548 0.7893 0.7517 0.7157 0.7157 0.7157 0.7157 0.7157 0.7157 0.7157 0.7157 0.7157	0.0 0.00000000000000000000000000000000	0.00.200.00.00.00.00.00.00.00.00.00.00.0	0.03159 0.03159 0.03159 0.03159 0.001131426322222222222222222222222222222222



I	J	К	Т(К)	THTD(K)	TDTD(K)	E(K)	GATE(K)
	023456789011234567890123456789010	13579135791357913579135791357913	0.0 12.000	106.6854 104.1513 102.3046 100.2979 98.7523 96.2384 93.7121 92.1999 87.1691 87.4691 83.4872 81.0928 76.7533 74.8176 71.3944 66.4753 73.1744 66.4753 74.8176 62.9476 62.04765 55.6025 55.6051 5	0.0930 -0.2112 -0.1789 -0.1729 -0.1545 -0.1849 -0.16857 -0.1849 -0.1703 -0.1703 -0.1703 -0.1735 -0.1735 -0.1735 -0.1737 -0.1737 -0.1739	-3.6582 0.1896 0.80552 0.80552 -1.1222 -0.83376 -2.64947 0.134980 -0.855328 1.34980 -0.6675 0.40675 -1.51270 -1.512827 0.1568842 -1.354363 -1.35436 -1.35	3600.088.33 5.84713.44.67.48.30.35 5.1275.88.30.35 4.638.30.30.30.30.30.99.33 4.0017078.33.30.30.99.30
112222222222222222222222222222222222222	023456789012345678901234567890 11111111112222222222223	2468024680246802468024680 1111122222233333344444555556	0.0 12.000	123.1348 123.14821 120.48605 118.2154 116.7974 115.70483 113.6861 119.9158 109.9158 109.96817 107.56817 107.56817 107.56817 107.455688 108.68498 109.91588 109.916888 109.91688 109.91688 109.91688 109.91688 109.91688 109.	-0.0544 -0.09552 -0.1637 -0.1259 -0.1160 -0.1160 -0.11200 -0.11335 -0.1212 -0.1297 -0.13362 -0.1430 -0.1430 -0.13527 -0.1430 -0.1442 -0.1442 -0.1471 -0.14671 -0.14671 -0.14671 -0.14671 -0.1500	-1.6245 -0.99668 -0.99668319 1.1009 0.656430 -0.27977 -0.57777 -0.57777 -0.57775 -0.47856683 -0.4785666 -0.88283 -1.882460 -2.88283 -1.882460 -2.88283	3600.022 8.384733 4.678873 4.638866 4.20546 4.20546 4.2057 4.309773 3.9933000 3.9933000 3.9933000 3.9933000 3.9932999999999999999999999999999999999



## TARGET NUMBER 1

## FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 1

K	FREO	PRF	PW	THETAD	THTD	SLAD	SLOD
13579135791357913579135791	1197.0 1197.0	00000000000000000000000000000000000000	00000000000000000000000000000000000000	104.4768 8155874844768 8150578748444768 8150578319868 80772319868 9077233098959459 90766299959411277 90766439187773.71.6643.618.6553.33.618.6553.33.618.6553.33.618.6553.33.618.6553.33.618.618.6553.33.618.618.618.618.618.618.618.618.618.618	104.131469 685131469 104.130479 100.2972841 100.2972841 986.271997631 986.271997631 988.91469728 887.1469763 887.1469767 887.1468774 887.1468774 887.1468774 887.1468774 887.1468774 887.1468774 887.1468774 887.1660 887.1660 887.1660 889.	1063069433091473006399003344452833333333333333333333333333333333	-118.5002 -118.5007 -118.5007 -118.50007 -118.50016 -118.50099 -118.50099 -118.50099 -118.50009 -118.500002 -118.500002 -118.500002 -118.500002 -118.500001 -118.49997 -118.49997 -118.49997 -118.49997 -118.49997 -118.50001 -118.50001 -118.50001 -118.50001 -118.50001 -118.50001 -118.50001 -118.50001 -118.50001

SMOOTHED INITIAL BEARING ANGLE = 106.68590 N FILTERED FINAL BEARING ANGLE = 50.60260 W

. VECTOR METHOD SOLUTION OF EMITTER LOCATION

.EMITTER LATITUDE = 33.22765 N

EMITTER LONGITUDE =-117.43607 W



#### TARGET NUMBER 2

#### FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER

2

К	FREQ	PRF	ΡΝ	THETAD	THTD	SLAD	SLOD
6 8 10	1212.0 1212.0	00000000000000000000000000000000000000	00000000000000000000000000000000000000	123.1348 122.48219 117.64229 117.64229 117.64223 114.8555 111.56732 119.4732 119.4732 1109.4731 109.4731 109.62317 102.2138 997.3126 94.384553 998.37126 94.38653 998.8788 88.	123.48 122.4821 120.9605 118.21574 115.7090 114.54856 115.7090 114.54856 111.6861 109.68158 107.95493 107.95493 107.95493 107.95493 107.976887 99.16887	32.9167 32.8835 32.8505 32.8174 32.7829 32.7166 32.6828 32.6162 32.5830 32.5498	-118.5004 -118.5004 -118.5004 -118.5004 -118.4997 -118.4999 -118.4999 -118.50002 -118.50001 -118.50001 -118.50001 -118.50007 -118.50007 -118.49995 -118.49995 -118.49995 -118.49999 -118.49999 -118.50002 -118.50007 -118.50007 -118.50007 -118.50002 -118.50007 -118.500007 -118.500007 -118.500007 -118.500007 -118.500007 -118.500007 -118.500007 -118.500007 -118.500007

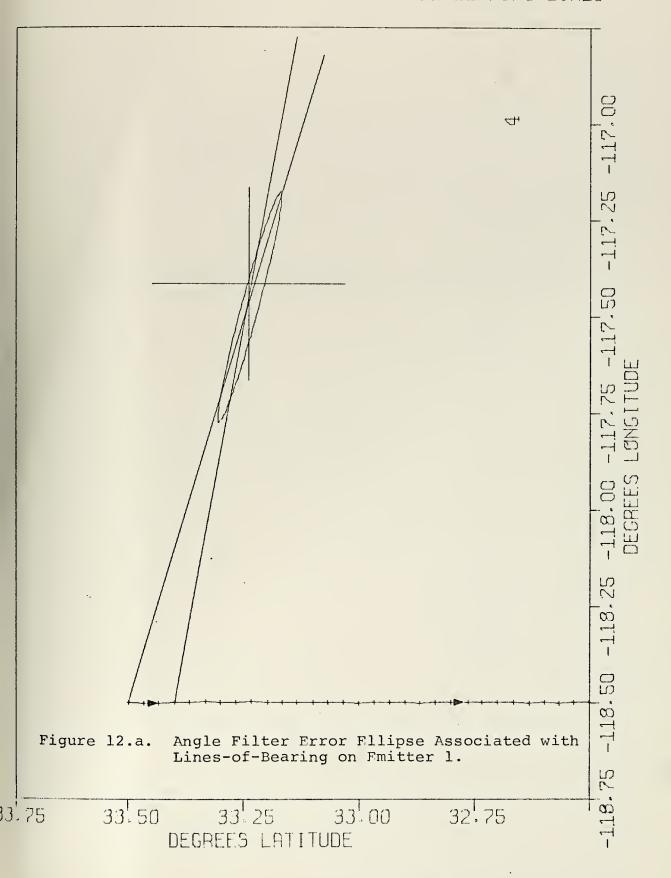
SMOOTHED INITIAL BEARING ANGLE = 123.13492 N FILTERED FINAL BEARING ANGLE = 75.94370 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 32.77921 N

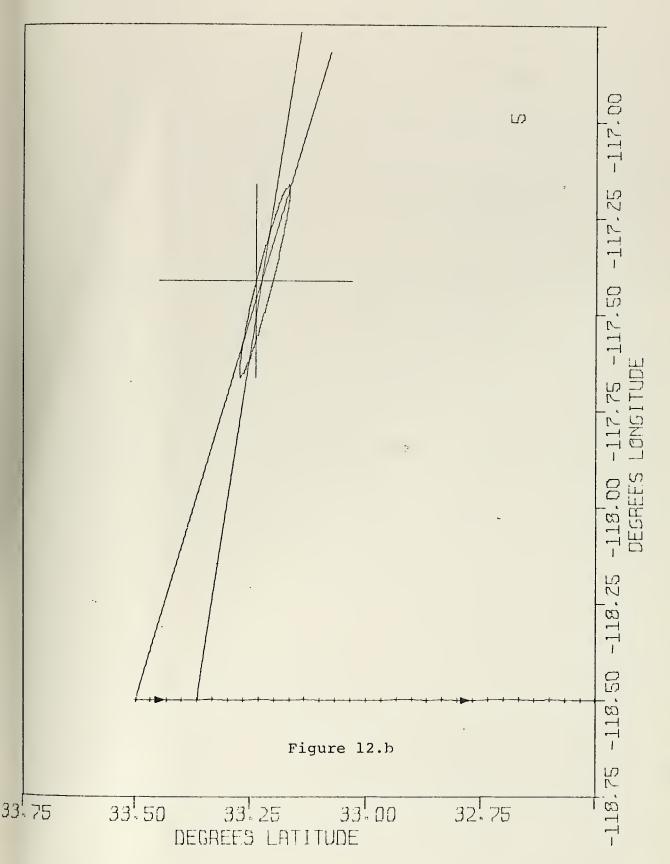
EMITTER LONGITUDE =-117.22505 W





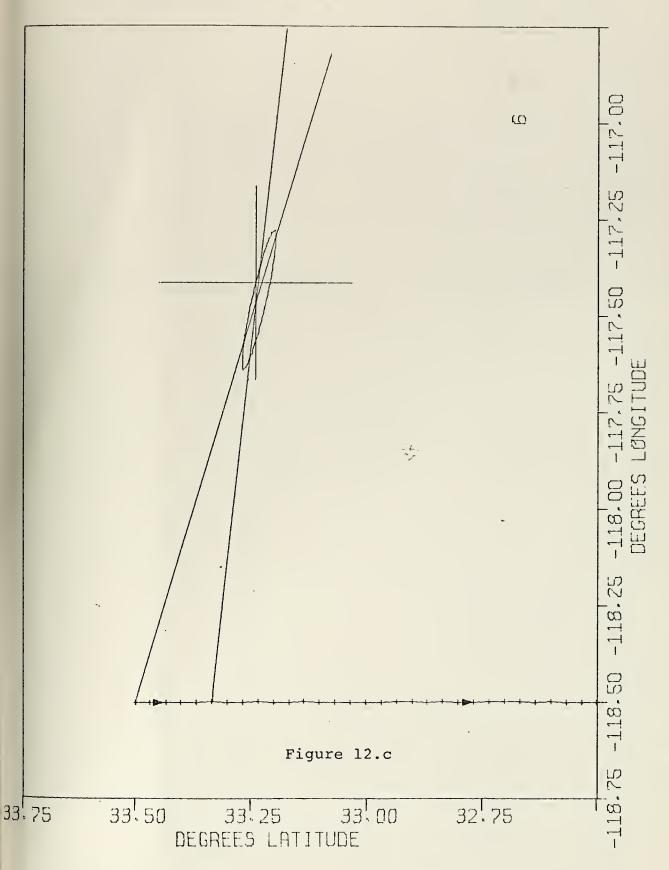


PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES



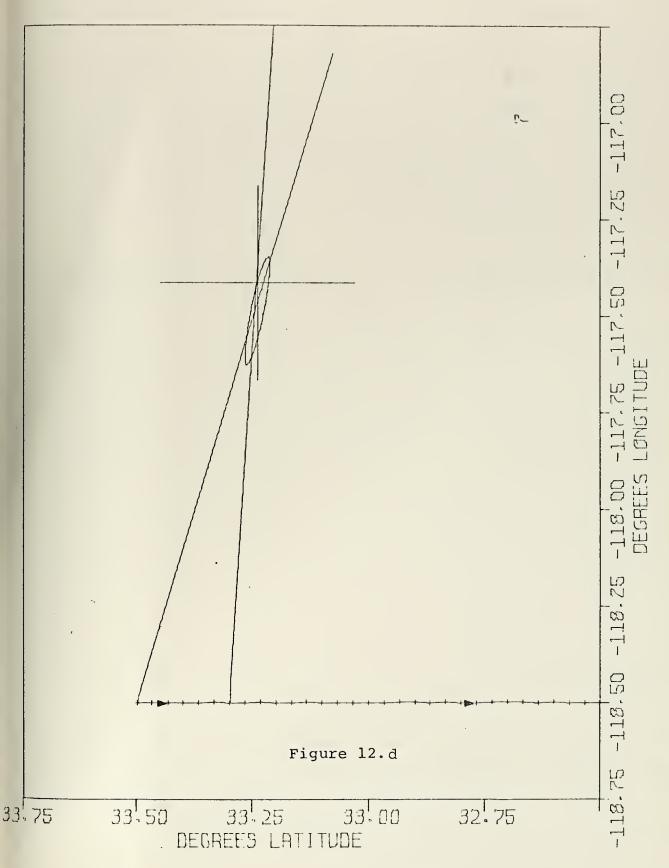


## PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES

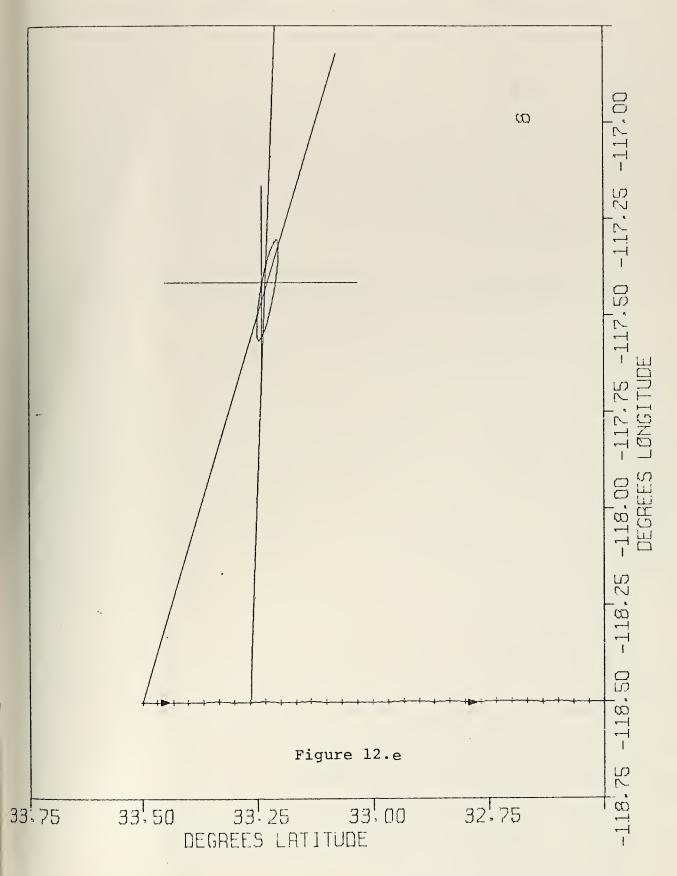




PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES

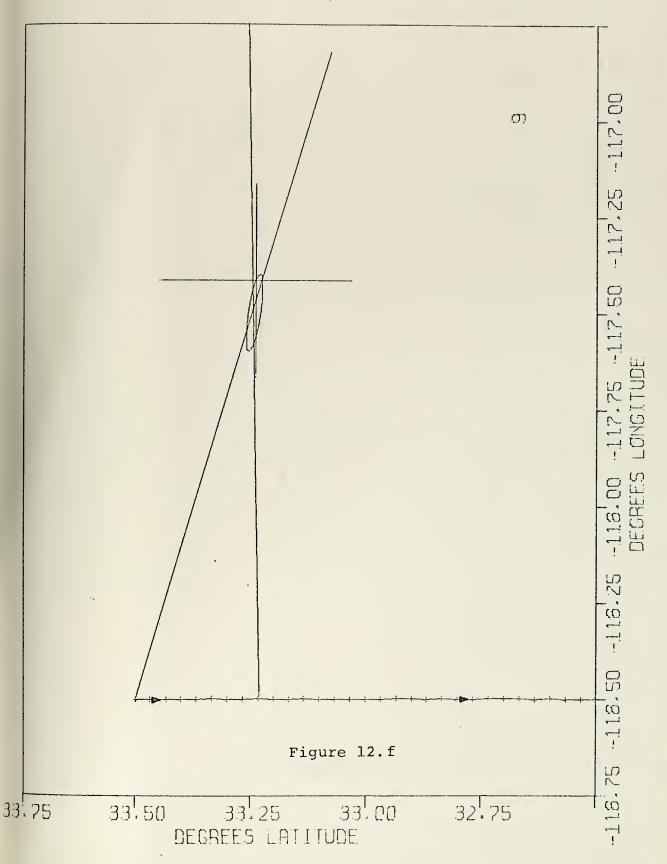






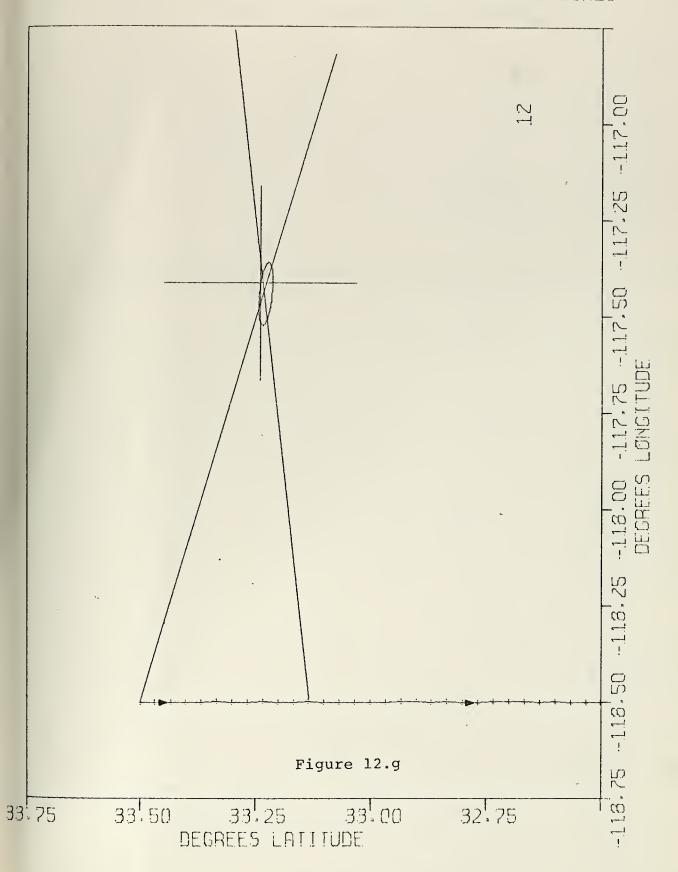


PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES



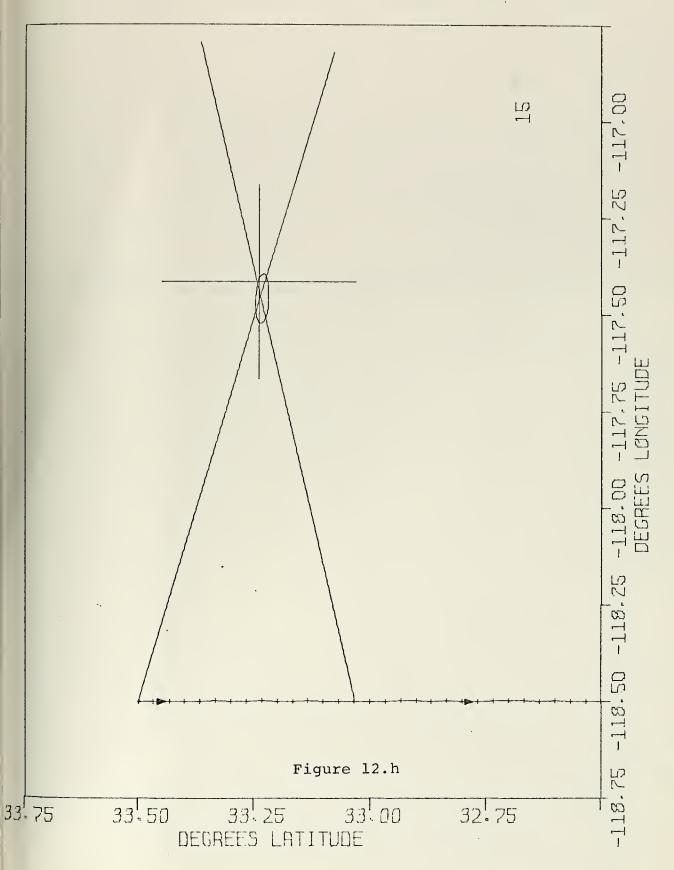


PLOT OF ERROR COURRIANCES OF SMOOTHED BEARING LINES



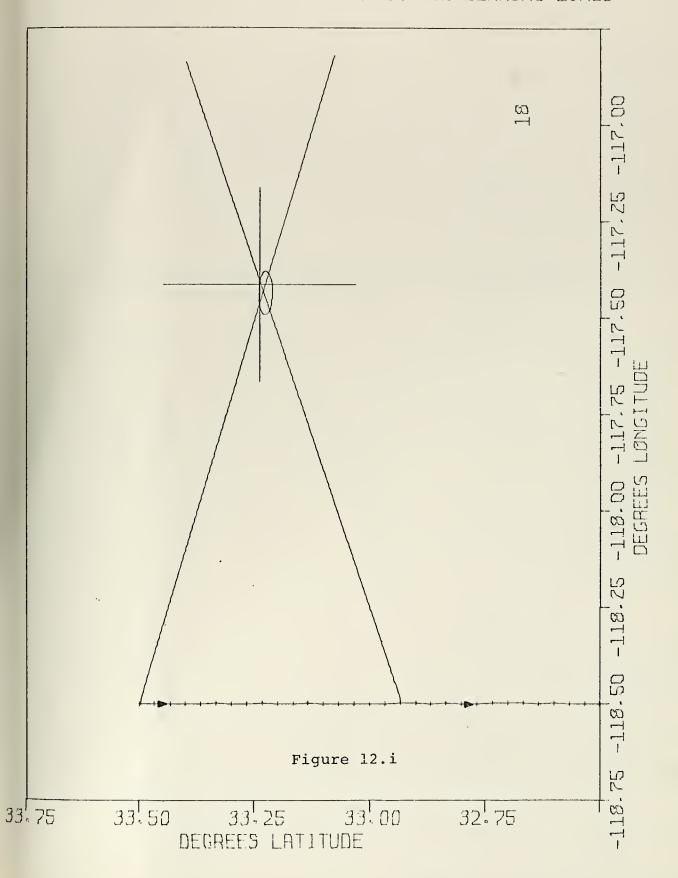


PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES

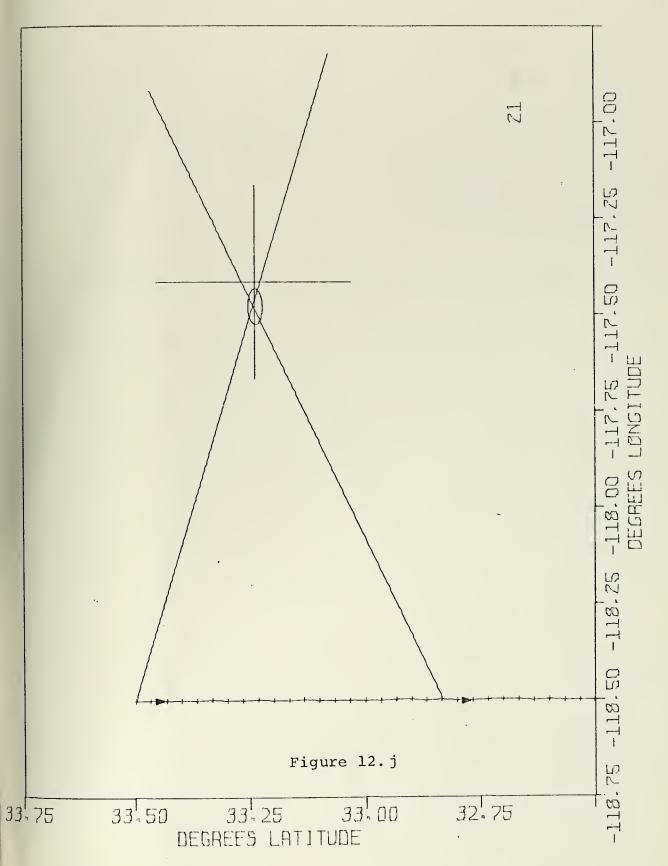




PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES

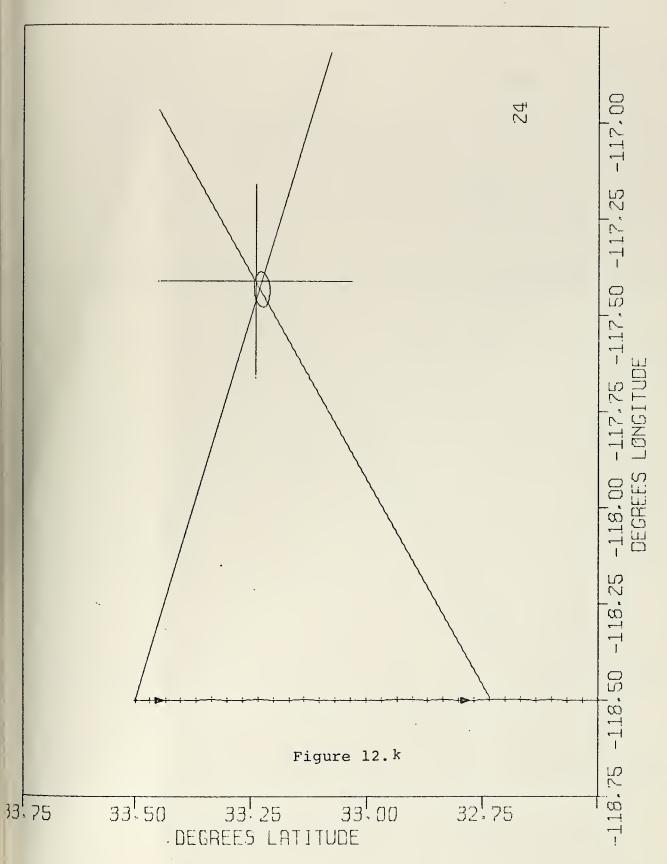






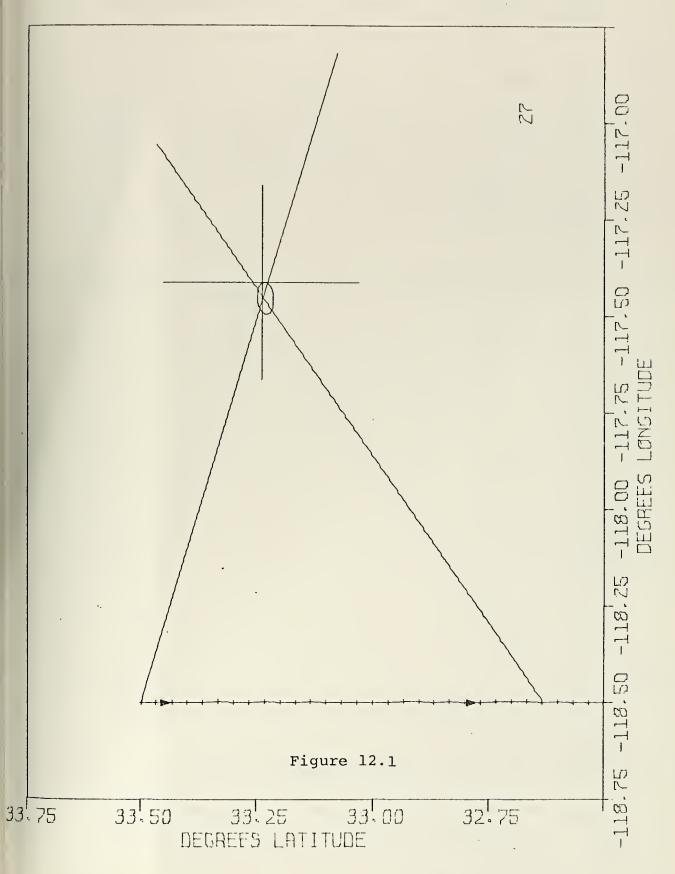


PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES

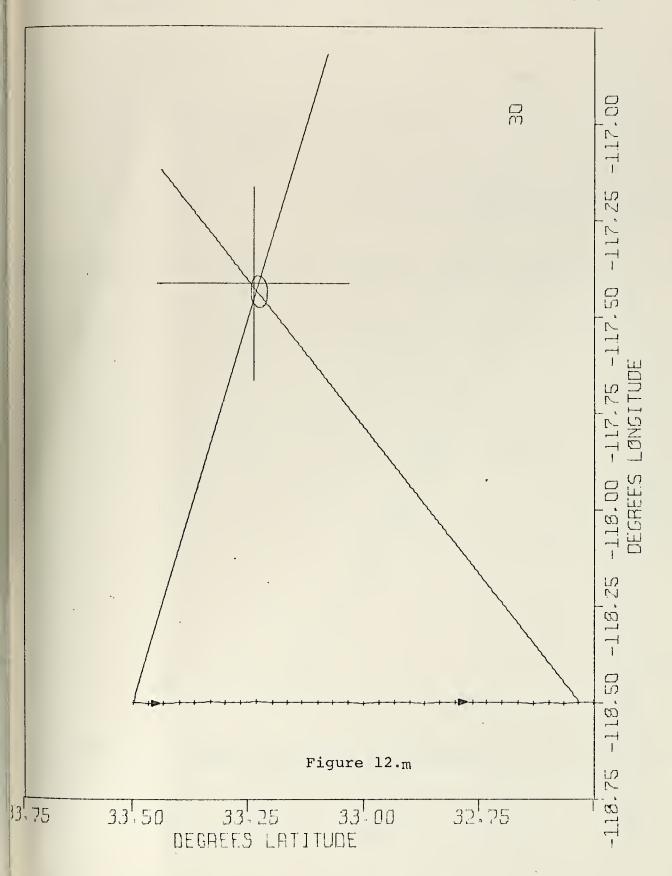




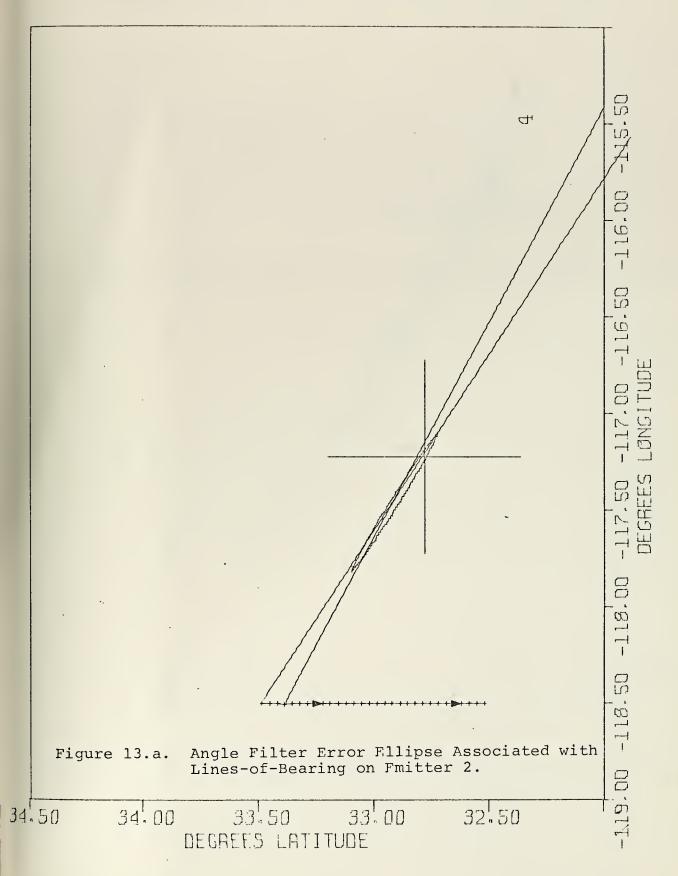
PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES



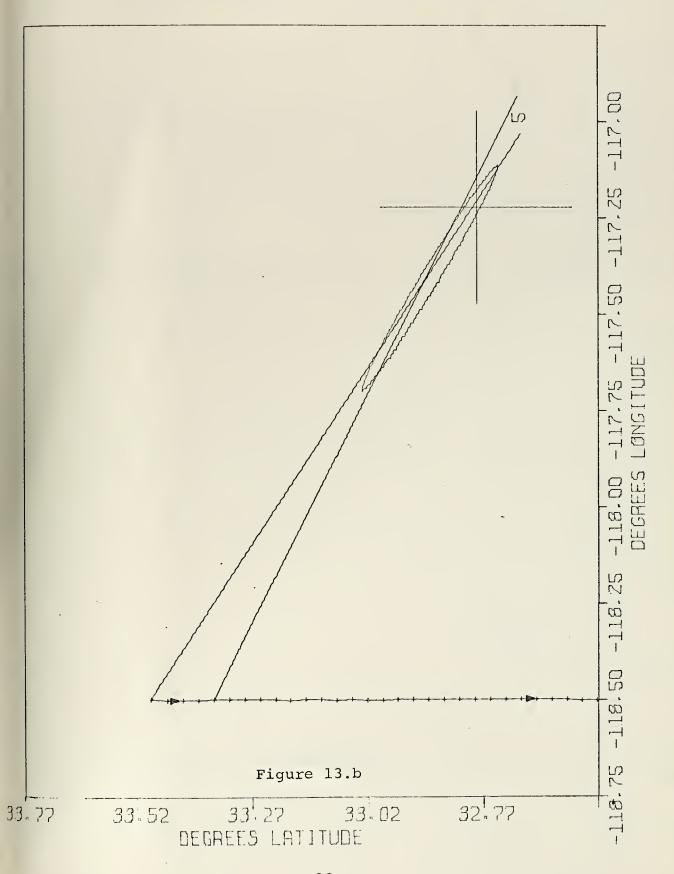




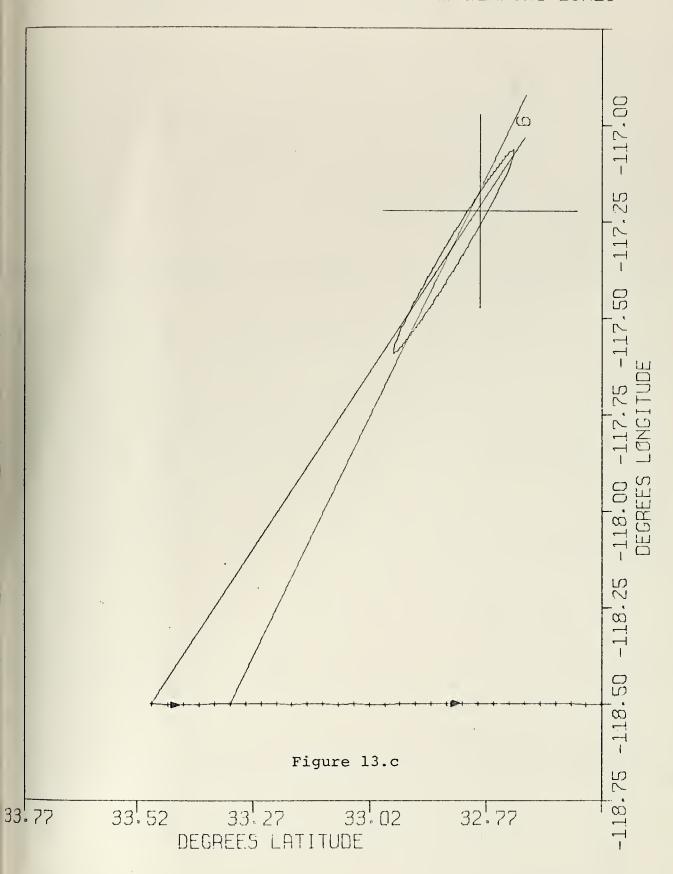




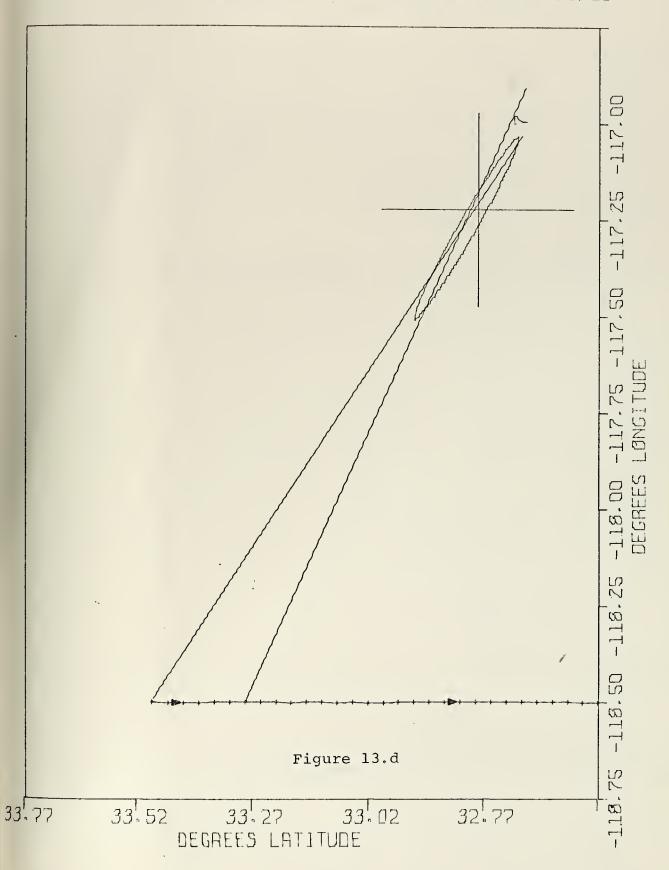




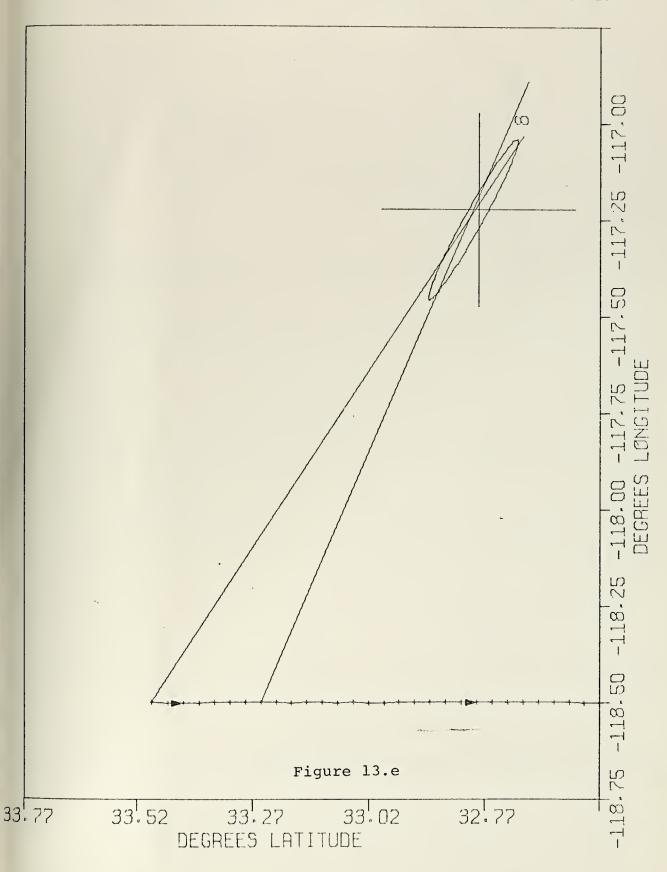




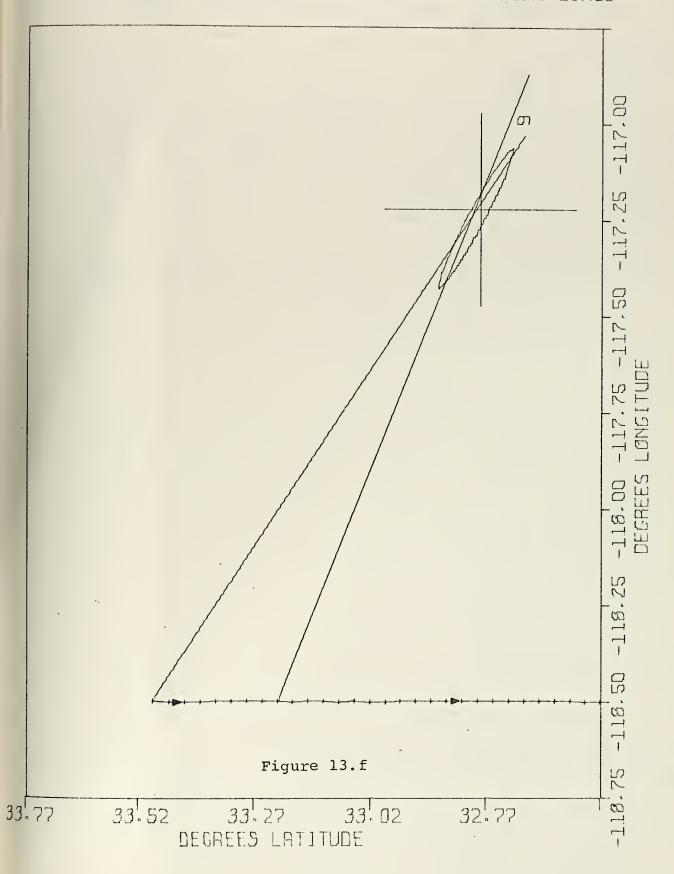




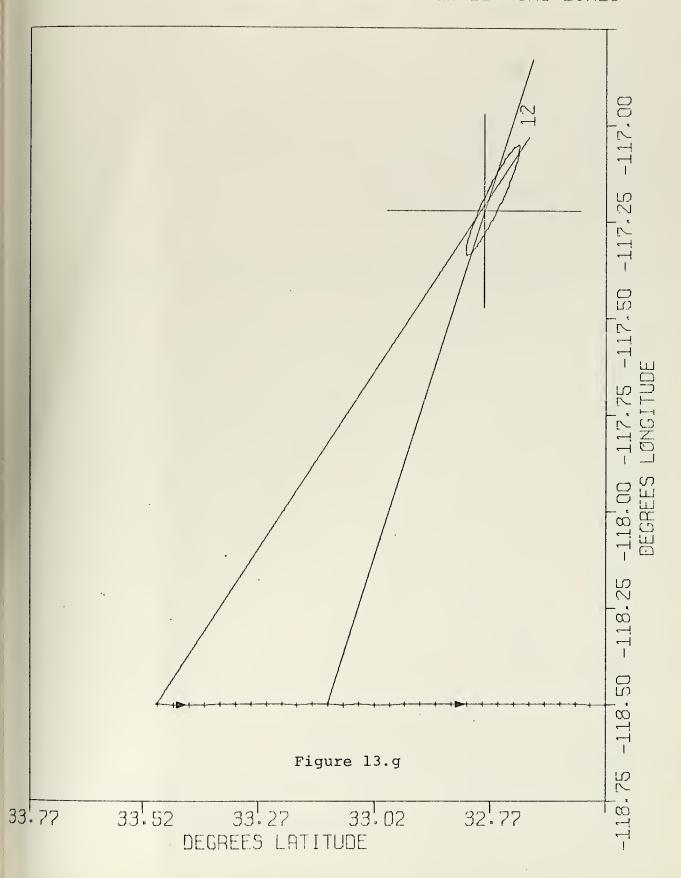




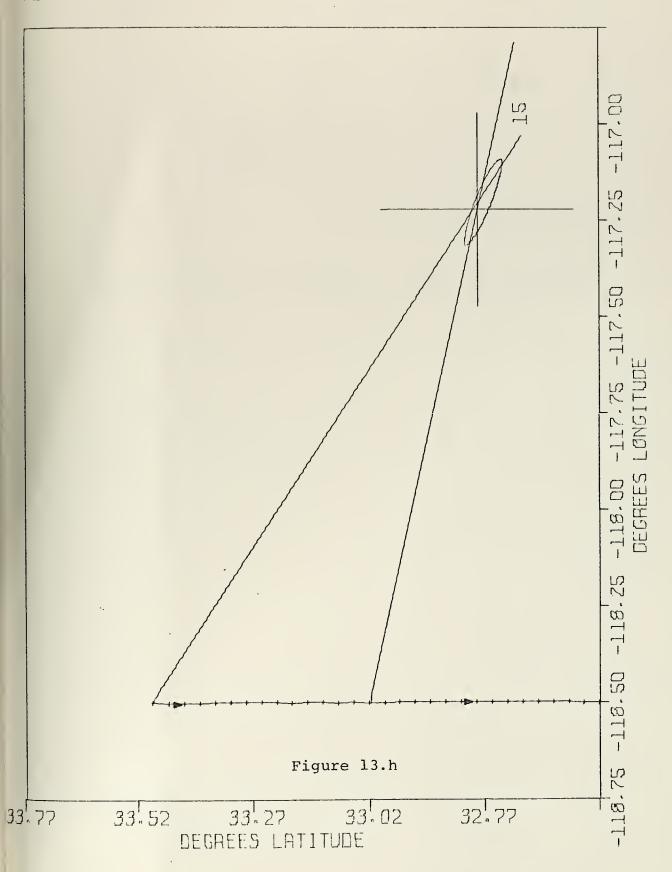




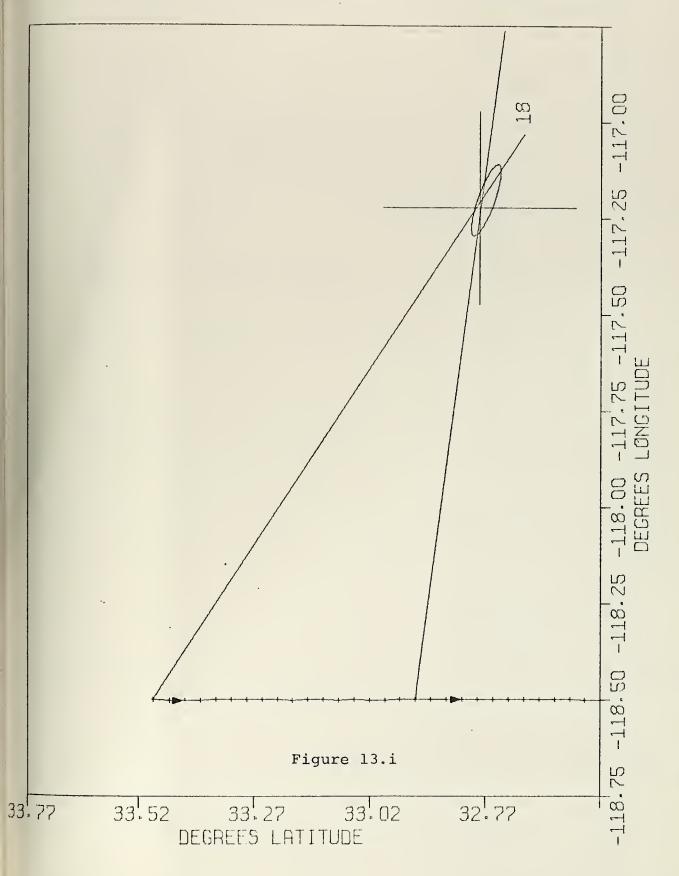






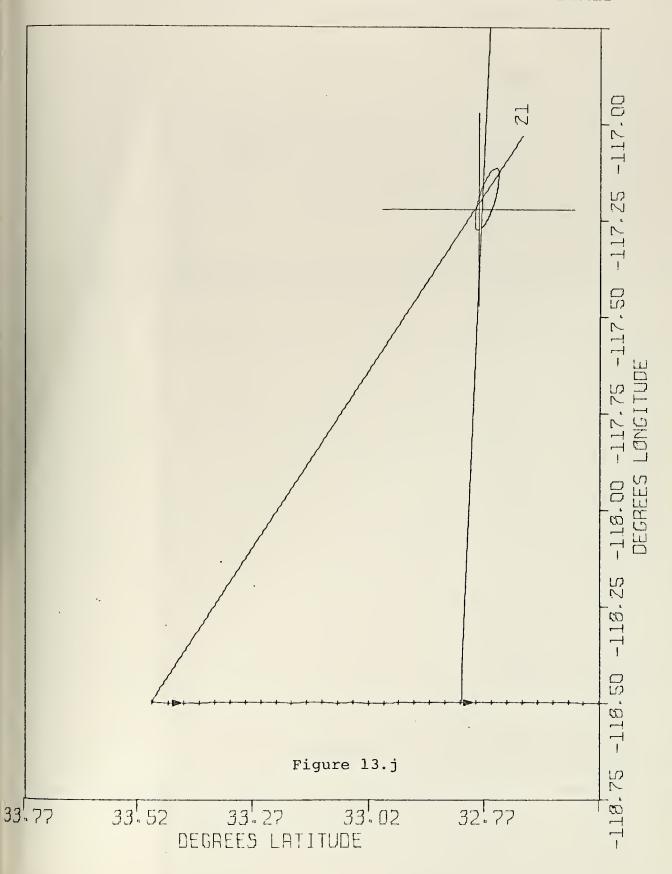






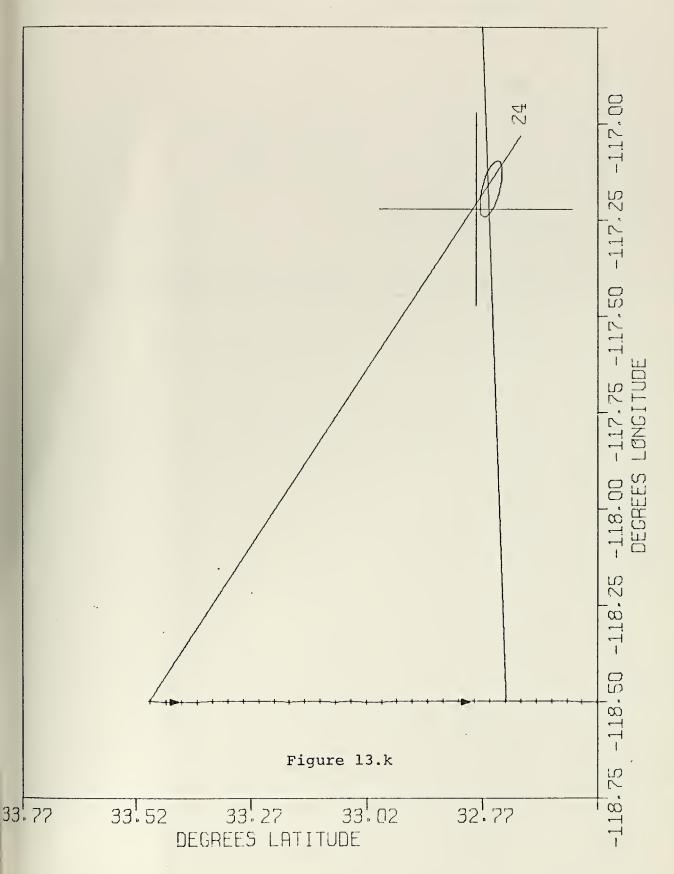


PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES

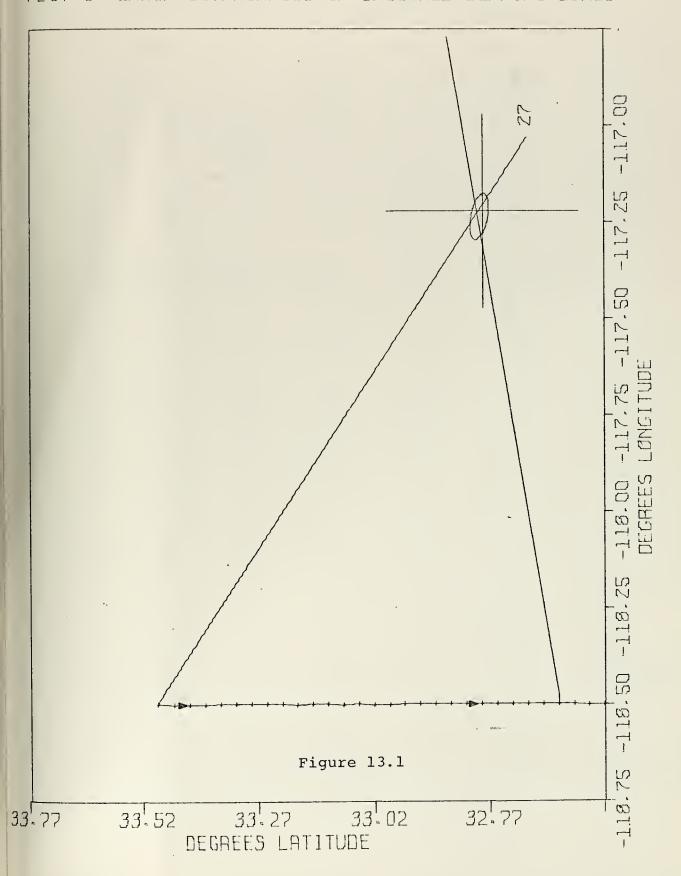




# PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES

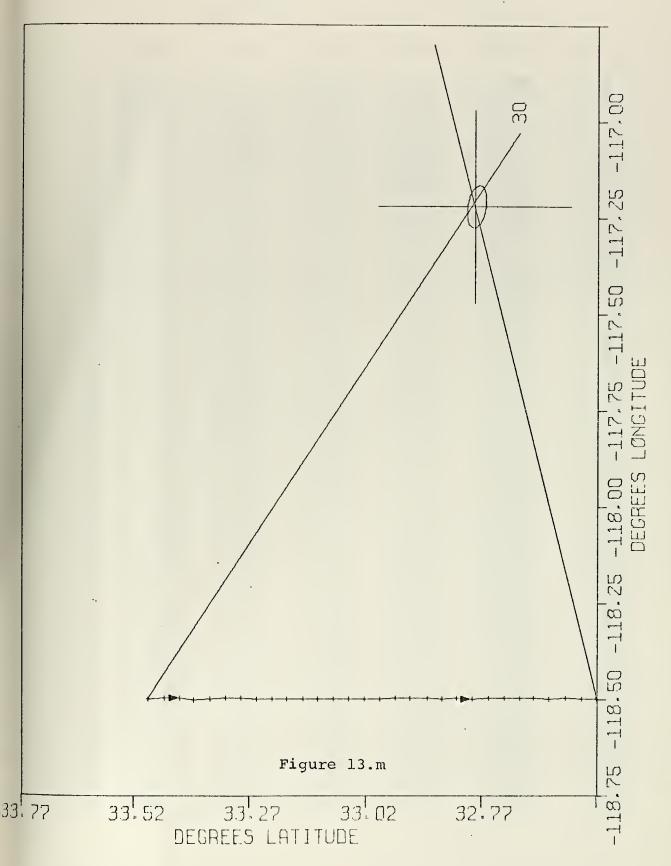








PLOT OF ERROR COVARIANCES OF SMOOTHED BEARING LINES





## NAVIGATION DATA FILTER PARAMETERS

K	SG1	SG2	SP 11	SP12	SP22	τ
12345678901234567890123456789012345678901234567890123456789012345678901	997725316666696969999999999999999999999999999	0.0 0.16216 0.16646 0.11174 0.10864 0.10903 0.10901	1.00000 37.00000 0.97585 0.00337 0.00261	0.12333333333333333333333333333333333333	1.00004 0.00004 0.00004 0.00008 0.000008	00000000000000000000000000000000000000



### NAVIGATION DATA FILTER PARAMETERS

К	VELND	ELAT	SLATD	VELED	ELON	SLOND
31 33 33 33 33 33 33 44 44 44 45	-0.00225 -0.00236 -0.00236 -0.00288 -0.00288 -0.002284 -0.002279 -0.002279 -0.002273 -0.002273 -0.002275 -0.002275 -0.002275 -0.002278 -0.002278 -0.002278 -0.002278 -0.002277	0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0	888886481388000040630502559957344764650589926632995527264880346826648803450074538064431583333333333333333333333333333333333	0.00003 -0.000012 -0.000012 -0.000012 -0.000016 -0.000016 -0.000016 -0.000016 -0.000016 -0.000016 -0.000001 -0.0000001 -0.0000001 -0.0000001 -0.0000001 -0.0000000 -0.0000000 -0.0000000 -0.0000000 -0.0000000 -0.000000000 -0.00000000 -0.00000000 -0.00000000 -0.00000000 -0.000000000 -0.0000000000	0.00088 0.00074 -0.00152 0.00033 -0.00065 0.00073 -0.00256 0.00053 -0.00012 -0.00135	-118.5009994 -118.5009924 -118.5009924 -118.500991628 -118.500991628 -118.500991628 -118.50099163 -118.5009914 -118.5009914 -118.5009914 -118.5009914 -118.5009914 -118.50099914 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.50099918 -118.5009918 -118.5009918 -118.5009918 -11



### KALMAN FILTER PARAMETERS FOR ANGLE FILTER

I	J	К	P11(K)	P12(K)	G1(K)	G2(K)
1 1 1 2 2 2	02340234	1357 2468	10000.0000 1440001.0000 6.8125 2.7990 10000.0000 1440001.0000 6.8125 2.7990	0.0 120000.0000 0.3202 0.1196 0.0 120000.0000 0.3202 0.1196	0.9999 1.0000 0.8720 0.7368 0.9999 1.0000 0.8720	0.0 0.0833 0.0410 0.0315 0.0 0.0833 0.0410 0.0315



### KALMAN FILTER PARAMETERS FOR ANGLE FILTER

I	J	К	T(K)	THTD(K)	TDTD(K)	E(K)	GATE(K)
1 1 1 2 2 2 2	0234 0234	1357 2468	0.0 12.000 12.000 12.000 0.0 12.000 12.000 12.000	106.6854 104.1513 102.3046 100.2979 123.1348 122.4821 120.9605 118.2154	0.0930 -0.2112 -0.1789 -0.1729 0.0810 -0.0544 -0.0952 -0.1637	-3.6504 0.7882 0.1896 -1.6245 -0.9966 -2.1748	3600.0022 8.3853 5.8473 3600.0022 8.3853 5.8473



THE EXTENDED KALMAN FILTER IS INITIATED WITH THE FOLLOWING ELLIPSE FOR CUT NUMBER 4 OF TARGET 1

THE ANGLE BETWEEN THE MAJOR AXIS AND THE MERIDIAN THROUGH THE CENTER OF THE ELLIPSE IS 104.941 DEGREES.

THE LENGTH OF THE SEMI-MAJOR AXIS IS

0.255 DEGREES.

THE LENGTH OF THE SEMI-MINOR AXIS IS

0.019 DEGREES.



К	P11	P12	Р	22	GX	GY	
79135791357913579135791	0.600E-0 0.587E-0 0.563E-0 0.563E-0 0.490E-0 0.440E-0 0.440E-0 0.355E-0 0.279E-0 0.221E-0 0.178E-0 0.178E-0 0.161E-0 0.164E-0 0.164E-0 0.115E-0 0.107E-0 0.107E-0 0.945E-0 0.893E-0 0.893E-0 0.893E-0	1 -0.154E- 1 -0.147E- 1 -0.137E- 1 -0.125E- 1 -0.125E- 1 -0.854E- 1 -0.854E- 1 -0.631E- 1 -0.541E- 1 -0.396E- 1 -0.295E- 1 -0.215E- 1 -0.159E- 1 -0.136E- 1 -0.16E- 2 -0.991E- 2 -0.991E- -0.	01 0.46 01 0.45 01 0.43 01 0.37 01 0.33 02 0.23 02 0.21 02 0.16 02 0.16 02 0.14 02 0.14 02 0.14 02 0.15 02 0.14 02 0.15 02 0.10 03 0.99 03 0.99 03 0.99 03 0.99 03 0.99	3E-02 3E-02 3E-02 7E-02 4E-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-02 8BE-03 8BE	0.112 0.213 0.2147 0.2247 0.22882 0.22882 0.22882 0.22337 0.2116 0.11597 0.1124 0.1124 0.01124 0.0987 0.00877 0.00877	-0.038 -0.0566 -0.0665 -0.081 -0.083 -0.083 -0.073 -0.063 -0.0564 -0.050 -0.042 -0.042 -0.030 -0.030 -0.029 -0.022 -0.023 -0.023	
K	DMX	DMY	TX	ER	x-	ΓD	YTD
79135791357913579135791	-0.1794 -0.1434 -0.1003 -0.0614 -0.0214 0.0184 0.0551 0.0958 0.1648 0.1974 0.2293 0.2598 0.3148 0.3348 0.33747 0.3928 0.4092 0.4227 0.44340 0.4516 0.4702 0.4738	-1.0314 -1.0706 -1.1082 -1.1383 -1.1676 -1.1950 -1.2111 -1.2323 -1.24498 -1.2532 -1.2548 -1.2524 -1.2524 -1.2524 -1.2130 -1.1646 -1.1643 -1.1032 -1.00810 -1.00360 -1.00137 -0.9917	1.7552 1.7169 1.6329 1.5921 1.55516 1.4762 1.44021 1.32935 1.22564 1.1273 1.1273 1.1277 1.09872 1.09872 1.09872 0.98665 0.9930 0.88696	-0.0038 0.0115 -0.0015 -0.0041 0.0299 -0.0279 0.0189 0.0189 0.0147 0.0028 0.0172 0.0295 0.0241 -0.0068 -0.0061 -0.0089 0.0142 -0.0089 0.0098 0.0098	-117.	4918 4890 48890 48890 48890 48890 48890 48700 447778 47778 47778 47701 466718 466718 46636	24476 333-24486 333-24486 333-24486 244879 244879 2244879 224455 224422 24433 23333 33333 33333 33333 33333 33333 33333



К	EP11	EP12	E	P22	G1	G2	
79135791357913579135791 11111122222333333444445555556	0.607E-01 0.239E-01 0.134E 00 0.106E 00 0.712E-01 0.538E-01 0.450E-01 0.387E-01 0.381E-01 0.385E-01 0.385E-01 0.385E-01 0.385E-01 0.385E-01 0.385E-01 0.385E-01 0.385E-01 0.385E-01 0.385E-01 0.385E-01 0.385E-01 0.385E-01	0.668E- 0.151E- 0.745E- 0.382E- 0.174E- 0.173E- 0.174E- 0.176E- 0.179E-	02 0.1 02 0.1	02223333333333333333333333333333333333	423897848733345555555555555555555555555555555	-0.145 0.082 0.082 0.0831 0.024 0.021 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220 0.0220	
K	EX	Ε <b>Y</b>	TODOTX	YTDDOT	ХT	D1	YTD1
27 29 31 33	0.0 -0.0054 -0.0006 -0.0019 -0.0018 0.0062 -0.0077 0.0062 0.0032 -0.0001 -0.0028 0.0021 0.0037 0.0024 -0.0020 -0.0015 -0.0036 0.0036 0.0036 0.0036 0.0036 0.0036 0.0036 0.0016 0.0036 0.0016 0.0009 0.0009 0.0020	0.0 -0.0059 -0.0034 -0.0011 -0.0005 -0.0021 -0.0022 -0.0026 -0.0014 -0.0002 -0.0010 -0.0007 -0.0012 -0.0008 0.0009 0.0008 0.0005 -0.0003 -0.0010 -0.0003 -0.0003 -0.0001 -0.0004 -0.0004	0.0010 0.0004 0.0003 0.0002 0.0001 0.0002 0.0002 0.0002 0.0002 0.0002 0.0002 0.0002 0.0002 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001 0.0001	0.0010 0.0005 0.0001 0.0001 0.0001 0.0000 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001 -0.0001	5 -117. 2 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117. 1 -117.	48899078695806239824589699355	33.3.2.24466 24477 33.3.2.24466 2244665 2244665 2244665 2244665 2244330 2244330 2244330 2244330 224400 224400 224400 22440 22440 233881 2333333333333333333333333333333333333



THE EXTENDED KALMAN FILTER IS INITIATED WITH THE FOLLOWING ELLIPSE FOR CUT NUMBER 4 OF TARGET 2

THE ANGLE BETWEEN THE MAJOR AXIS AND THE MERIDIAN THROUGH THE CENTER OF THE ELLIPSE IS 122.129 DEGREES.

THE LENGTH OF THE SEMI-MAJOR AXIS IS 0.361 DEGREES.

THE LENGTH OF THE SEMI-MINOR AXIS IS 0.023 DEGREES.



К	P11	P12	Р	22	GX	GY	
8024680246802468024680 111112222223333333444445555556	0.934E-0 0.933E-0 0.932E-0 0.9925E-0 0.906E-0 0.870E-0 0.816E-0 0.669E-0 0.589E-0 0.513E-0 0.335E-0 0.2956E-0 0.256E-0 0.1256E-0 0.143E-0 0.1143E-0 0.118E-0 0.109E-0 0.925E-0	01 -0.583E- 01 -0.582E- 01 -0.582E- 01 -0.564E- 01 -0.564E- 01 -0.459E- 01 -0.356E- 01 -0.356E- 01 -0.225E- 01 -0.165E- 01 -0.165E- 01 -0.164E- 01 -0.921E- 01 -0	01 0.37 01 0.36 01 0.36 01 0.34 01 0.25 01 0.25 01 0.19 01 0.16 01 0.16 01 0.16 01 0.16 01 0.55 01 0.55 01 0.33 01 01 0.33 01 0.33	2E-01 1E-01 0E-01 0E-01 0E-01 01E-01 01E-01 05E-01 04E-01 04E-01 04E-01 04E-01 04E-01 04E-01 04E-01 04E-01 04E-02 04E	0.038 0.038 0.136 0.1193 0.2317 0.3370 0.3370 0.3370 0.32942 0.2270 0.1280 0.1180 0.1180 0.1180 0.1180 0.1180	0.015 -0.032 -0.083 -0.176 -0.209 -0.242 -0.242 -0.243 -0.204 -0.184 -0.160 -0.146 -0.134 -0.114 -0.1057 -0.097 -0.098 -0.073 -0.064	
К	DMX	DMY	TX	ER	X-	ΓD	YTD
8024680246802468024680 112222223333333444445555556	-0.3977 -0.3825 -0.3630 -0.3630 -0.3208 -0.2977 -0.2727 -0.2477 -0.2212 -0.1952 -0.1699 -0.1481 -0.0936 -0.0677 -0.0433 -0.0189 0.0066 0.0315 0.0546 0.0782 0.1687 0.1456 0.1908 0.2102	-0.6036 -0.6431 -0.6822 -0.7220 -0.7603 -0.7959 -0.8287 -0.8595 -0.8901 -0.9131 -0.9326 -0.9517 -0.96876 -1.00461 -1.0175 -1.06647 -1.07571 -1.06647 -1.07571 -1.0873 -1.0930 -1.0984 -1.1013	2.06793 4.06793 4.09794446 1.09794446 1.09794446 1.09794446 1.0979446 1.0979446 1.0979446 1.09794 1.09	-0.0139 0.002112 0.02314 0.02314 0.02315 0.02658 0.0498 0.0581 0.05444 0.0591 0.0517 0.0517 0.0517 0.0407 0.03673 0.0198 0.0133	-117, -117,	4464	32.9048 905528 32.900332 900528 900538 900528 900528 800775 8882417 888248 8



К	EP11	EP12	EP	22	G1	G2	
8024680246802468024680 111122222333333444468024680	0.234E 0	0.00 0.6025E- 0.225E- 0.225E- 0.225E- 0.3304E- 0.171E- 0.1774E- 0.1779E- 0.1799E- 0.1799E- 0.1799E- 0.1799E- 0.1799E- 0.1799E- 0.179E-	01 0.13 01 0.22 02 0.47 02 0.17 02 0.15 02 0.16 02 0.16	5370298033333333333333333333333333333333333	00000000000000000000000000000000000000	-0.407 0.187 0.079 0.043 0.028 0.022 0.020 0.019 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020 0.020	
K	EX	EY	TODOTX	YTDDOT	X <sup>-</sup>	TD1	YTD1
8024680246802468024680 111122222333333444468024680	0.0 -0.0046 -0.0004 0.0017 0.0041 0.0059 0.0072 0.0063 0.0154 0.0054 0.0054 0.0054 0.0054 0.0054 0.0054 0.0054 0.0054 0.0054 0.0054 0.0054 0.0054 0.0054 0.0057 -0.0030 0.0057 -0.0037 -0.0002	0.0 -0.0062 0.0008 -0.0012 -0.0030 -0.0041 -0.0049 -0.0036 -0.0051 -0.0053 -0.0015 -0.0023 0.0040 -0.0040 -0.0040 -0.0040 -0.0040 -0.0040 -0.0040 -0.0040 -0.0040 -0.0040 -0.0044 -0.0047 0.0045	0.0010 0.0001 0.0001 0.0002 0.0003 0.0004 0.0004 0.0006 0.0008 0.0008 0.0008 0.0008 0.0008 0.0008 0.0006 0.0006 0.0006 0.0006 0.0005 0.0005 0.0003 0.0003	0.0010 -0.0001 -0.0001 -0.0003 -0.0004 -0.0005 -0.0009 -0.0011 -0.0011 -0.0011 -0.0011 -0.0011 -0.0011 -0.0011 -0.0011 -0.0010 -0.0010 -0.0010	-117 -117 -117 -117 -117 -117 -117 -117	802 44438188 4444333688 444438188 444438188 44438188 44438188 44438188 444438188 444438188 444438188 444438188 44438188 444738188 444738188 444738188 444738188 444738188 444738188 444738188 444738188 444738188 444738188 444788188 444788188 444788188 444788188 444788188 444788188 444788188 4447888 444788 444788 44488 4	2.9046 32.90999 32.89999 32.89999 32.89999 32.88909 32.89909 32.8909 32.8909 32.8909



#### TARGET NUMBER 1

#### FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 1

K	FREQ	PRF	PW	THETAD	THTD	SLAD	SLOD
<u>3</u>	1197.0 1197.0 1197.0	150.0 150.0	3.50	106.6854 104.1513 102.4055 100.3478	106.6854 104.1513 102.3046 100.2979	33.4670 33.4326	-118.5004 -118.5002 -118.5007 -118.5012

SMOOTHED INITIAL BEARING ANGLE = 106.68600 N FILTERED FINAL BEARING ANGLE = 100.29791 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 33.24323 N

EMITTER LONGITUDE =-117.49606 W

EXTENDED KALMAN FILTER SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 33.23833 N

EMITTER LONGITUDE =-117.45799 W



#### TARGET NUMBER 2

#### FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 2

K	FREQ	PRF	PW	THETAD	THTD	SLAD	SLOD
4	1212.0 1212.0 1212.0 1212.0	250.0 250.0	3.00	123.1348 122.4821 120.8329 117.6429	123.1348 122.4821 120.9605 118.2154	33.4498 33.4160	-118.5002 -118.5004 -118.5004 -118.5015

SMOOTHED INITIAL BEARING ANGLE = 123.13493 N FILTERED FINAL BEARING ANGLE = 118.21536 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 32.90477 N

EMITTER LONGITUDE =-117.44853 W

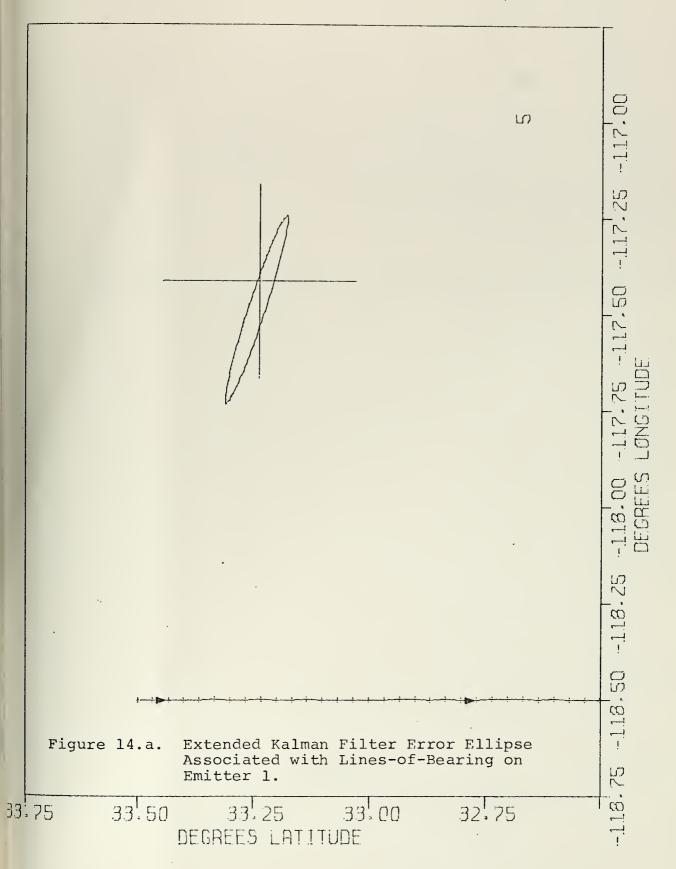
EXTENDED KALMAN FILTER SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 32.77782 N

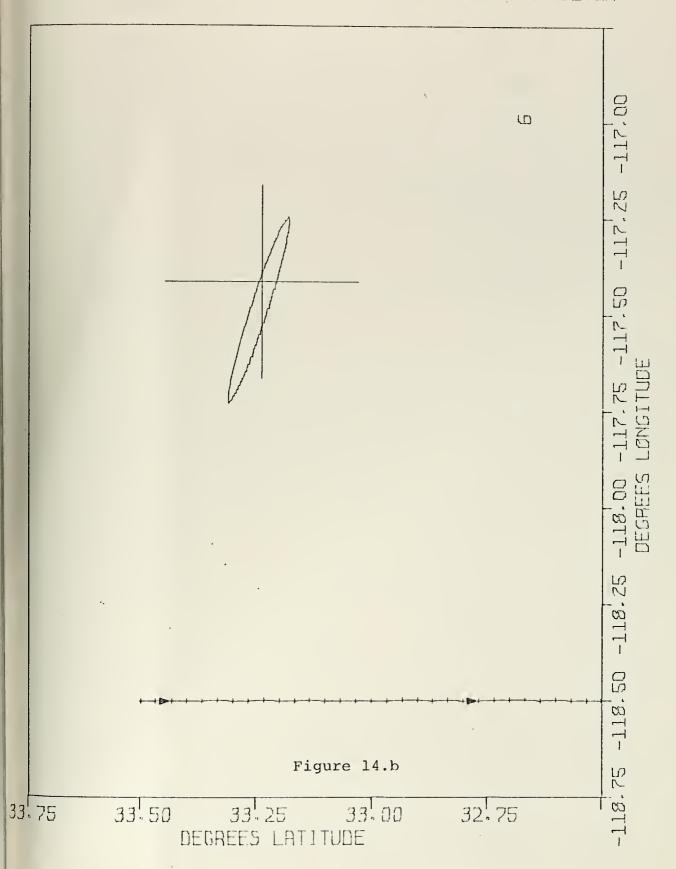
EMITTER LONGITUDE =-117.31906 W



PLOT OF ERROR COVARIANCES OF EXTENDED KALMAN FILTER

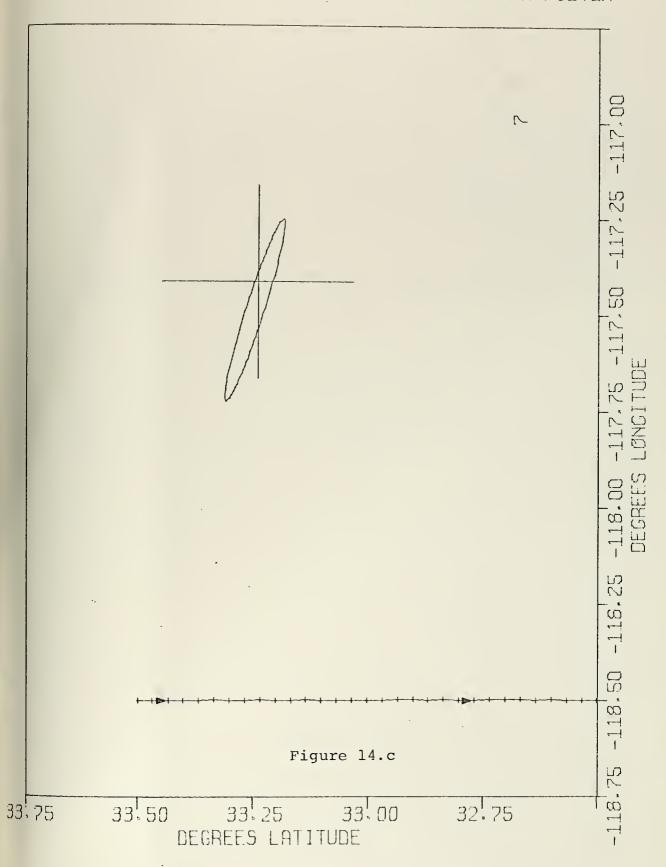




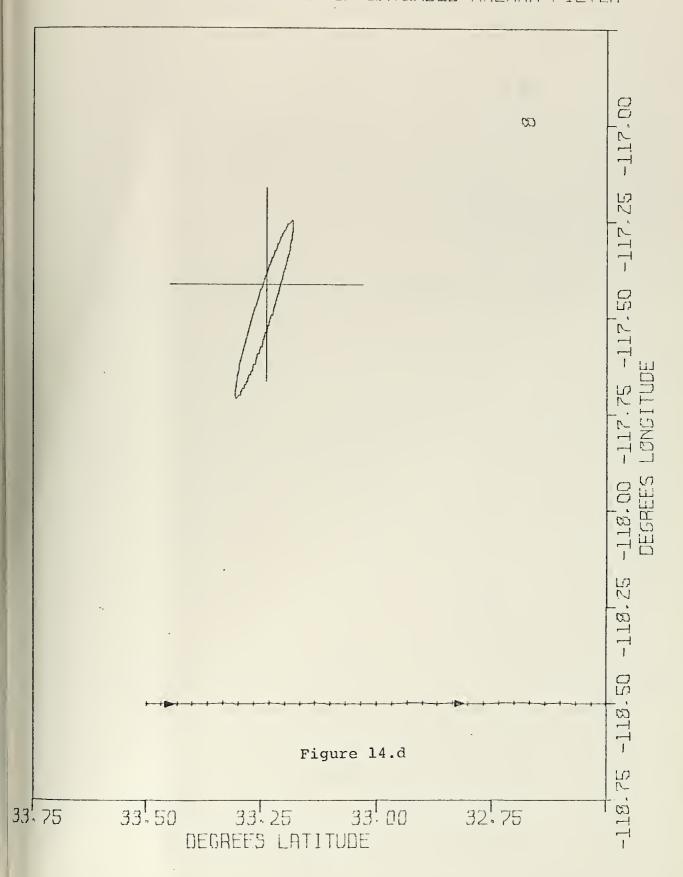




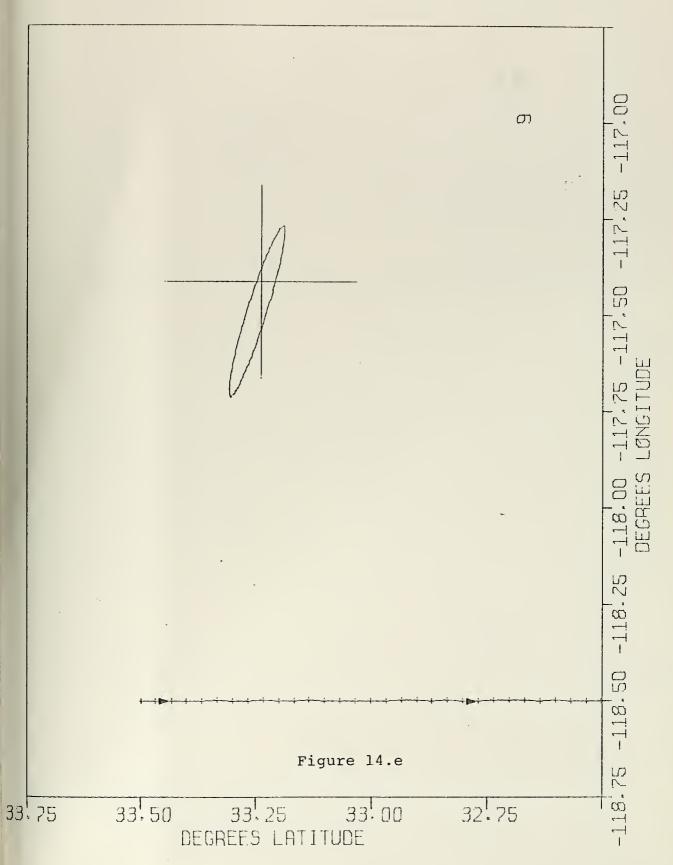
PLOT OF ERROR COVARIANCES OF EXTENDED KALMAN FILTER



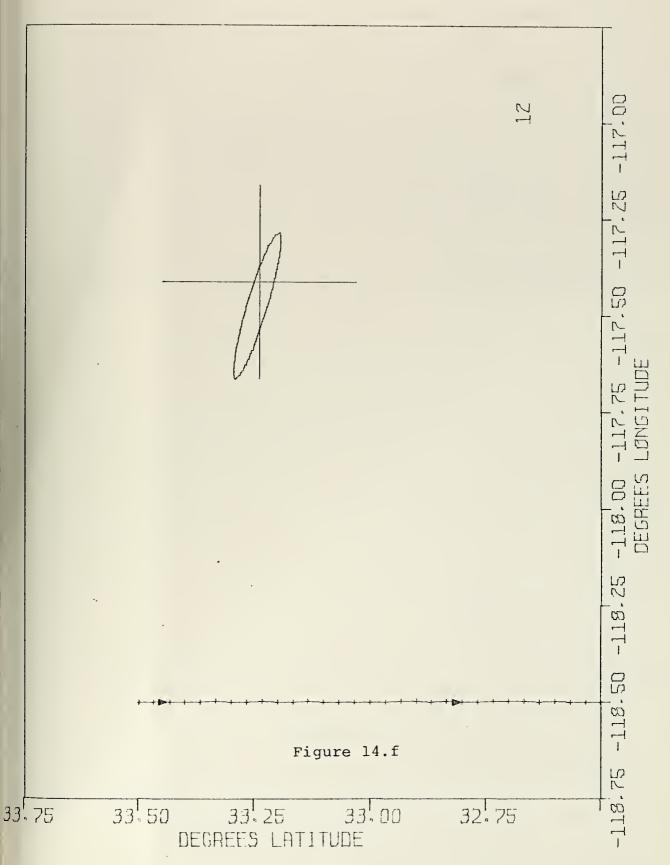




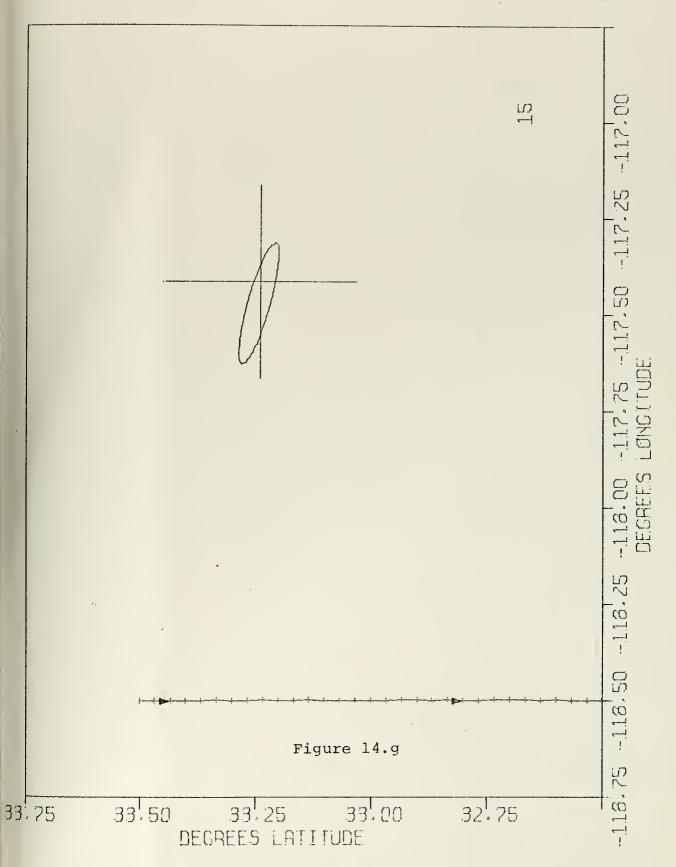




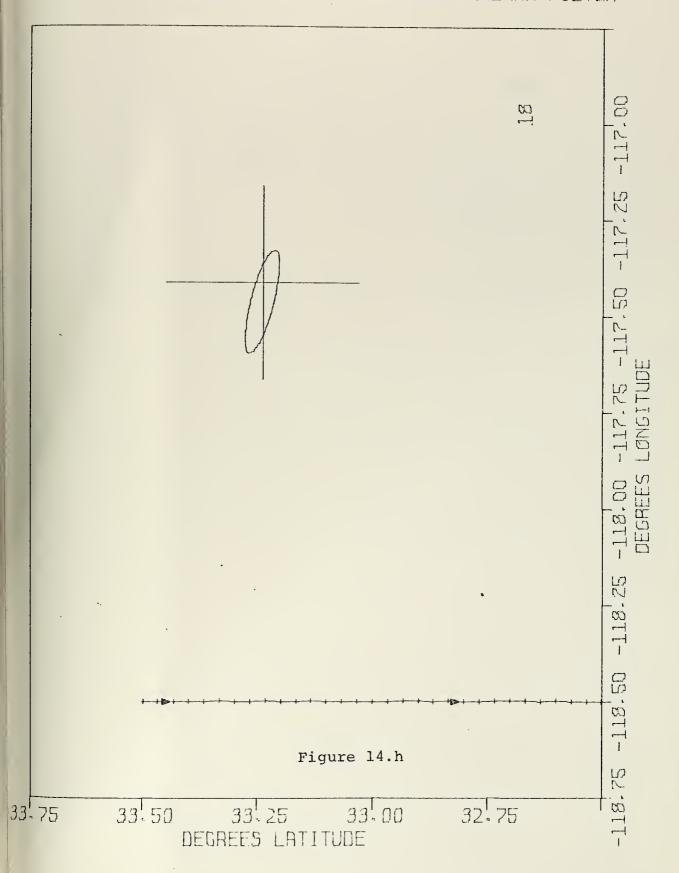




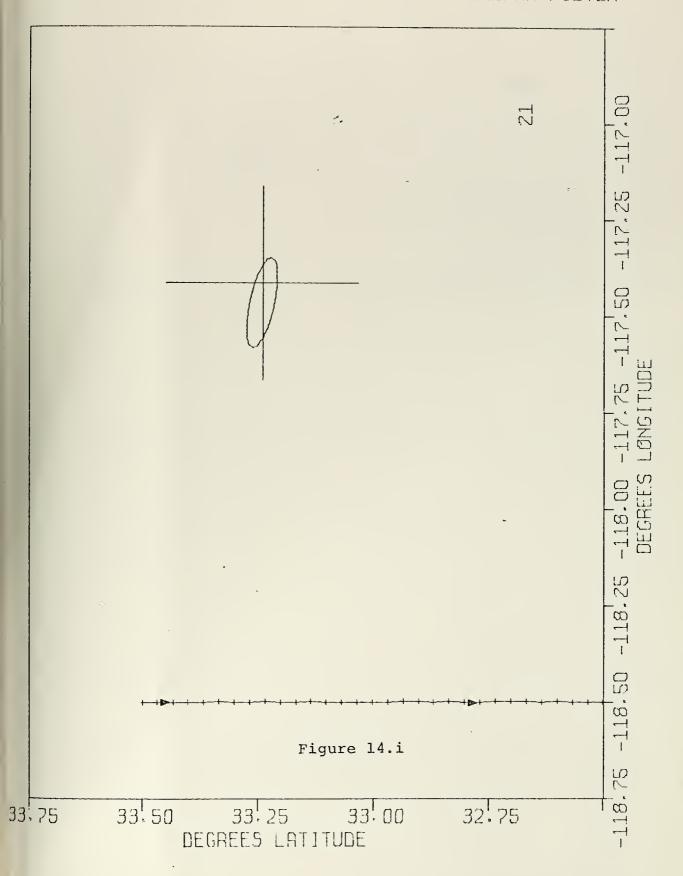




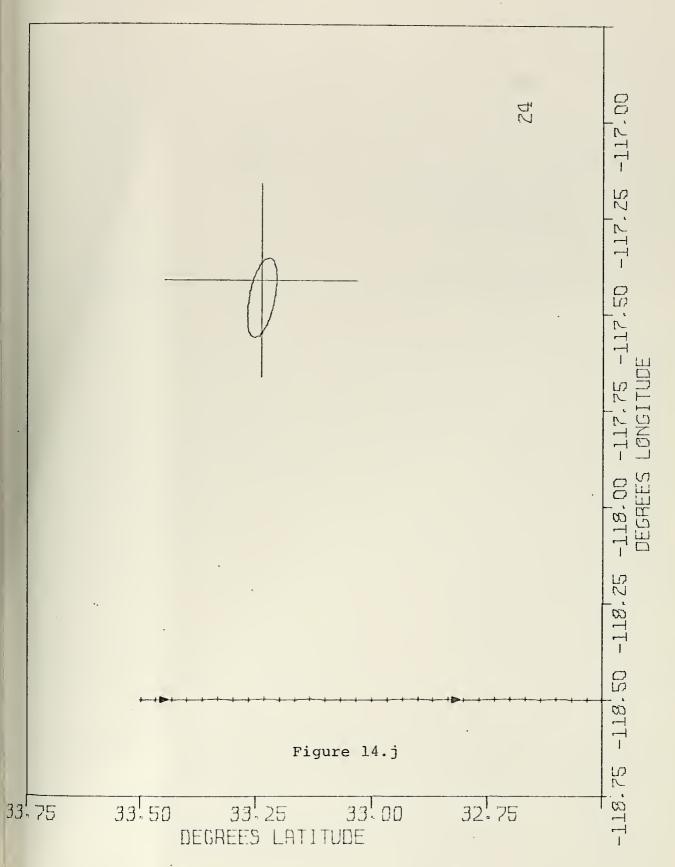




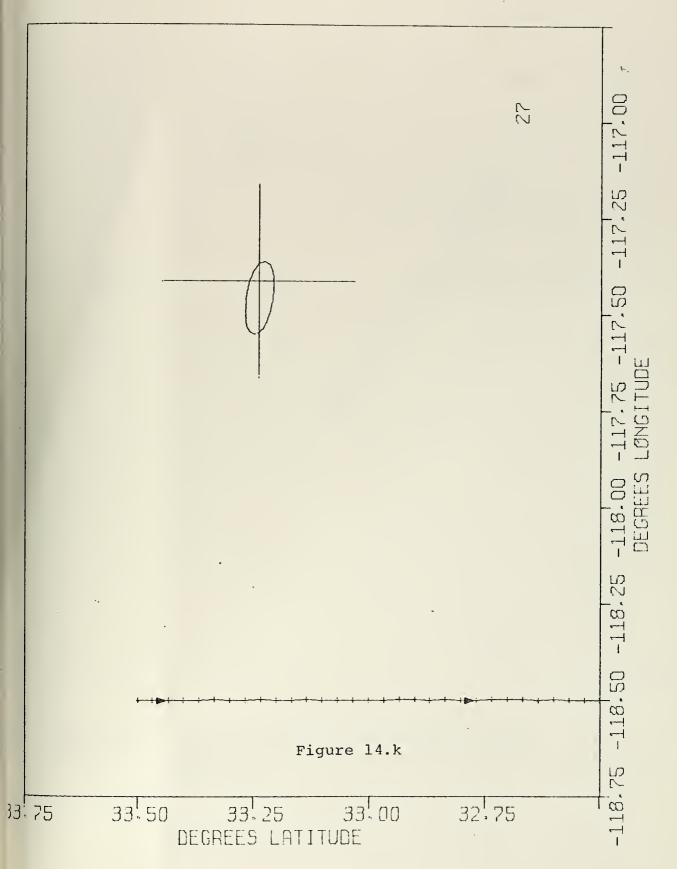




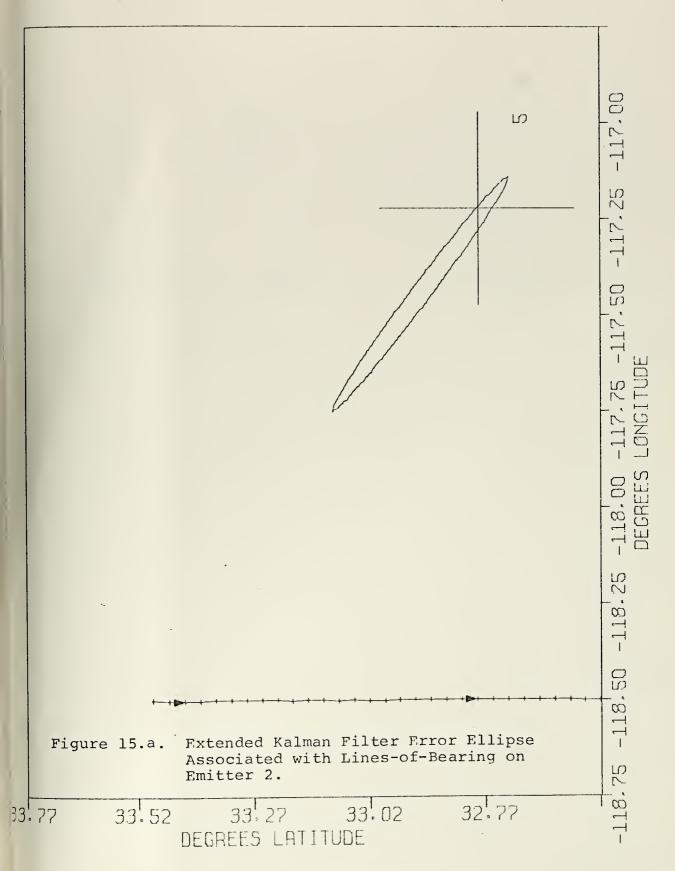




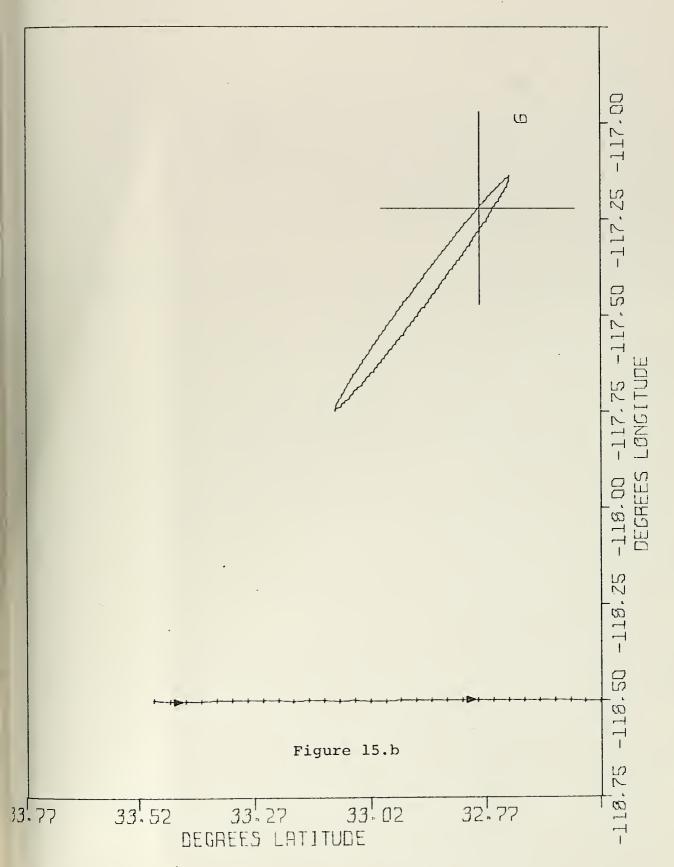




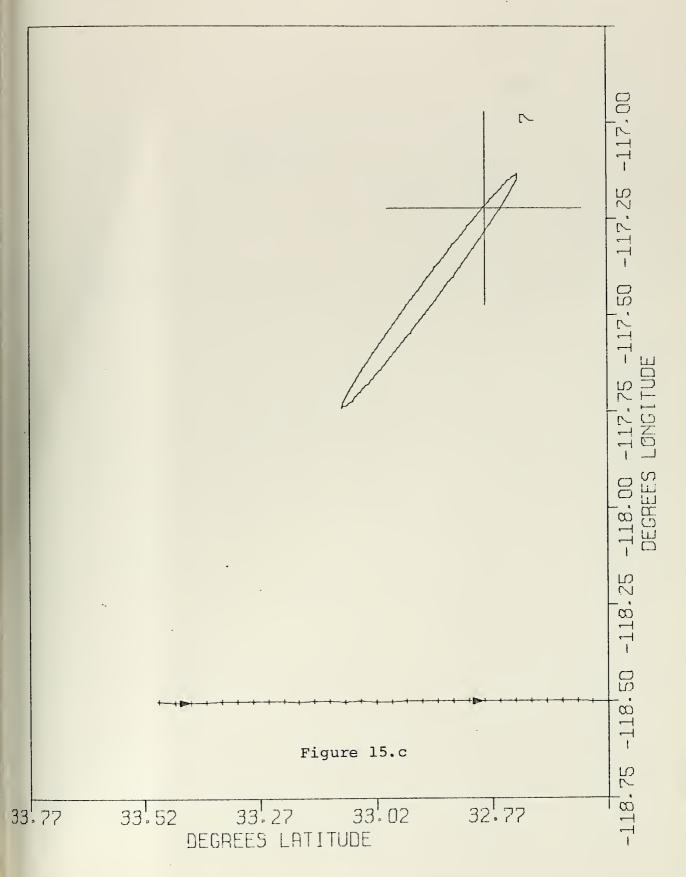




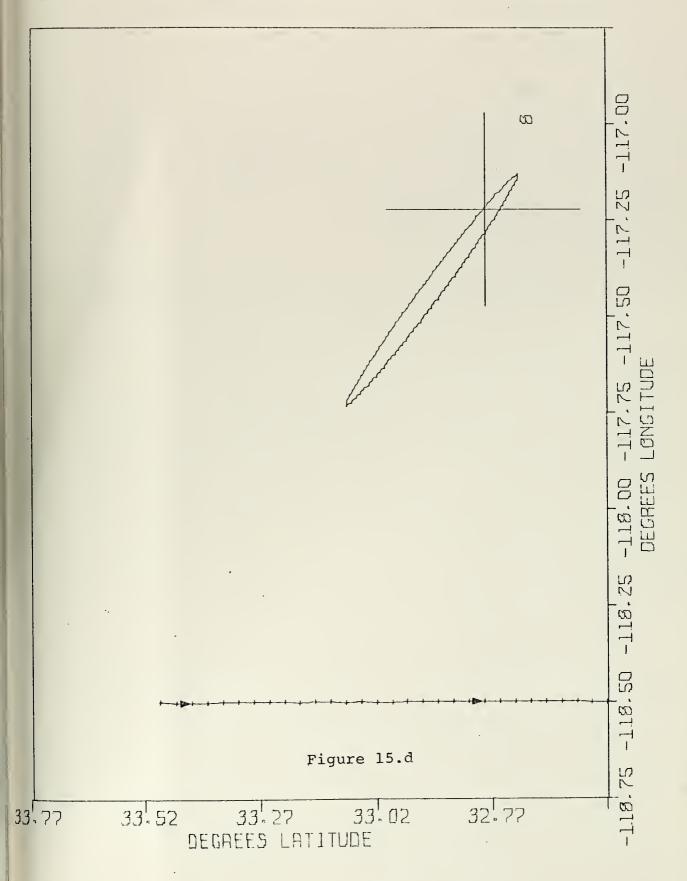






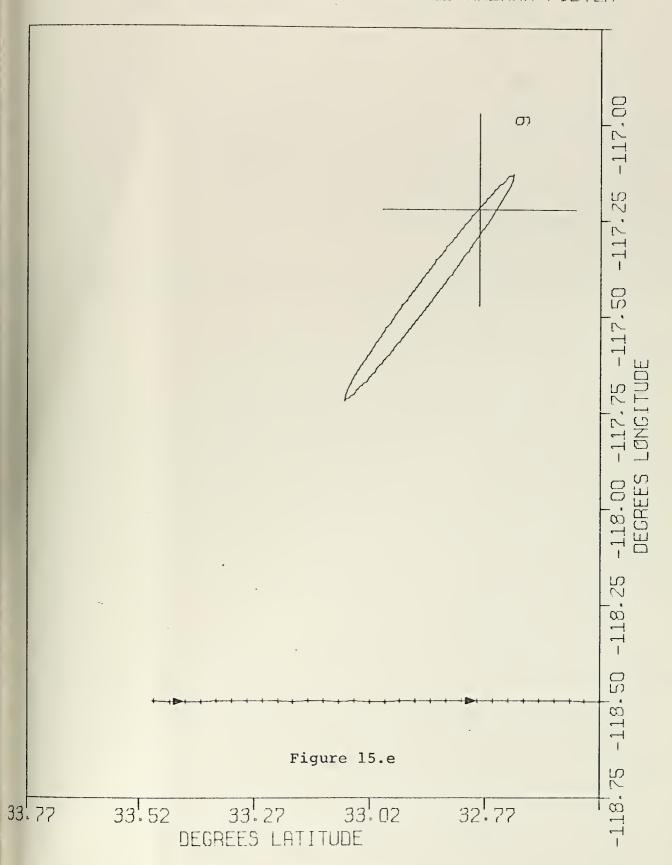




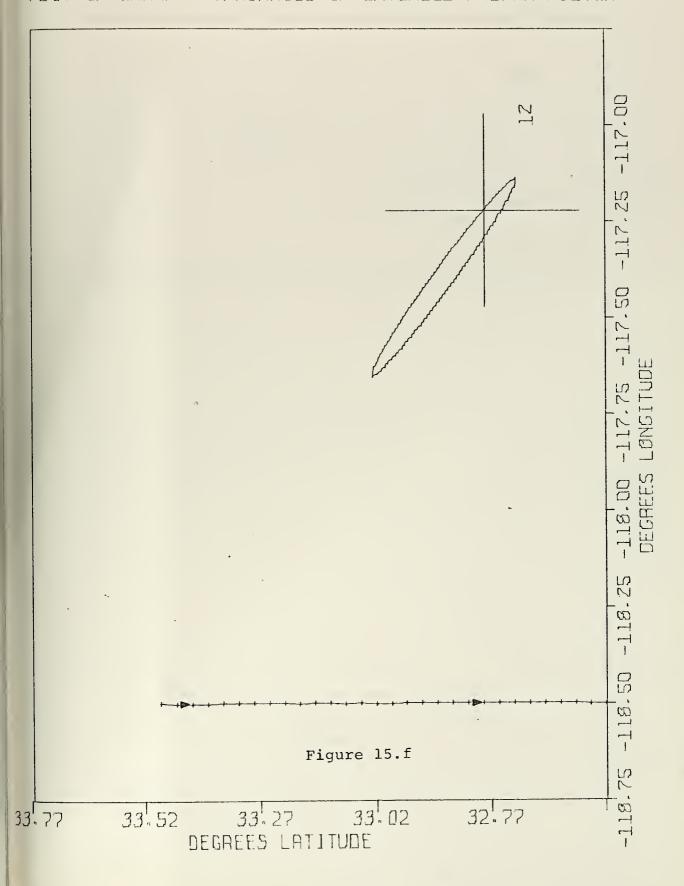




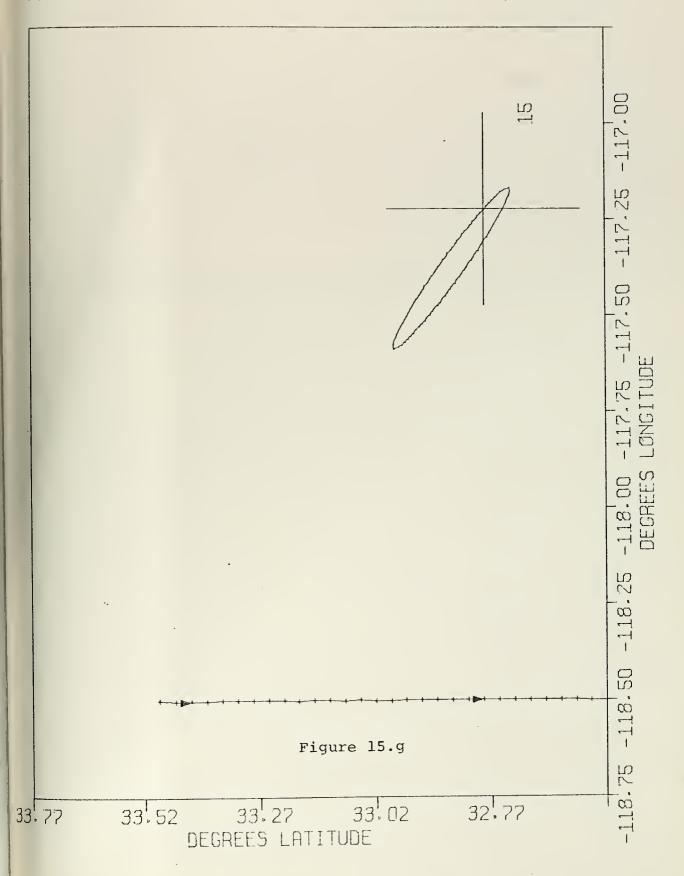
PLOT OF ERROR COVARIANCES OF EXTENDED KALMAN FILTER



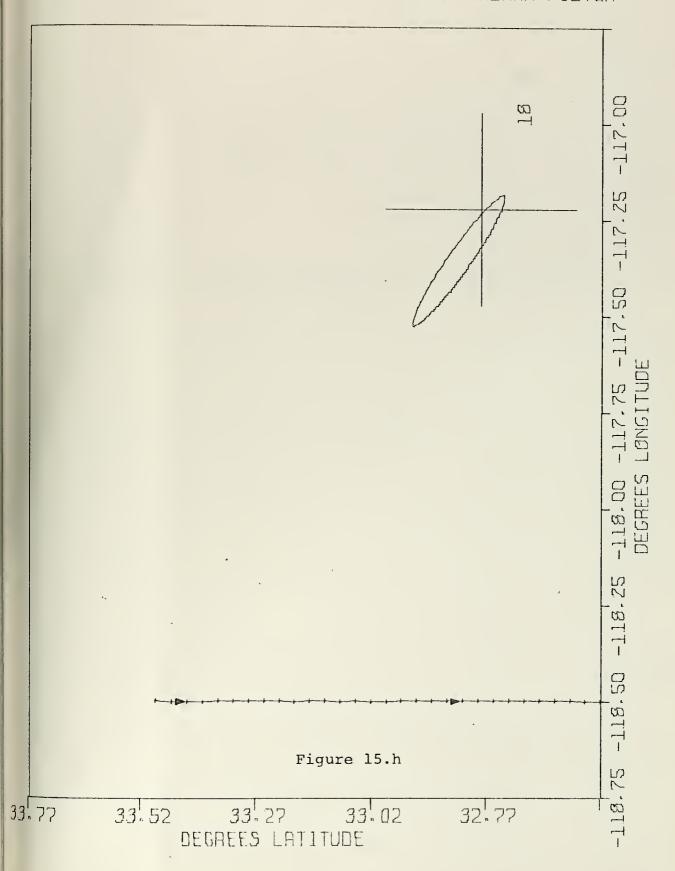




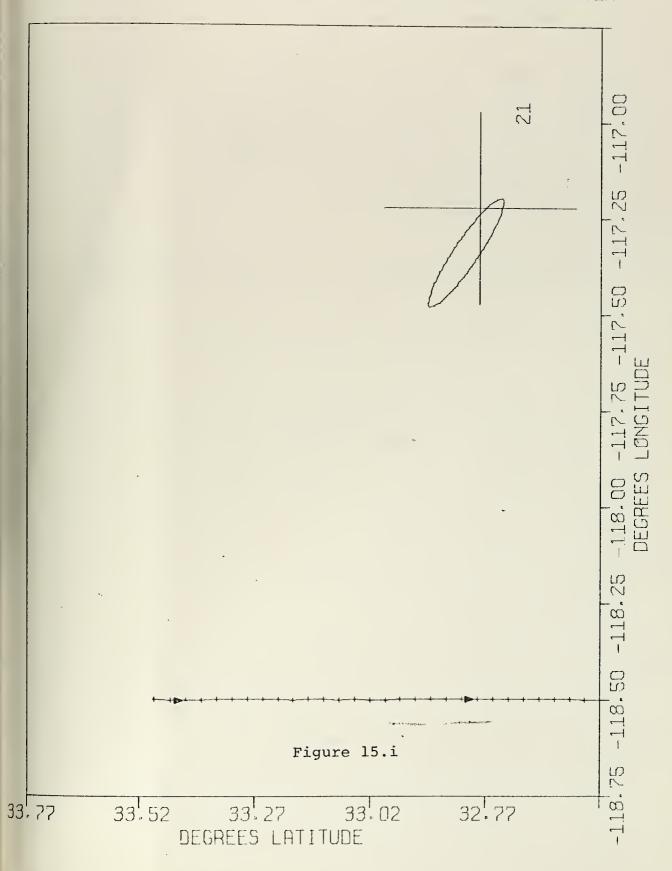






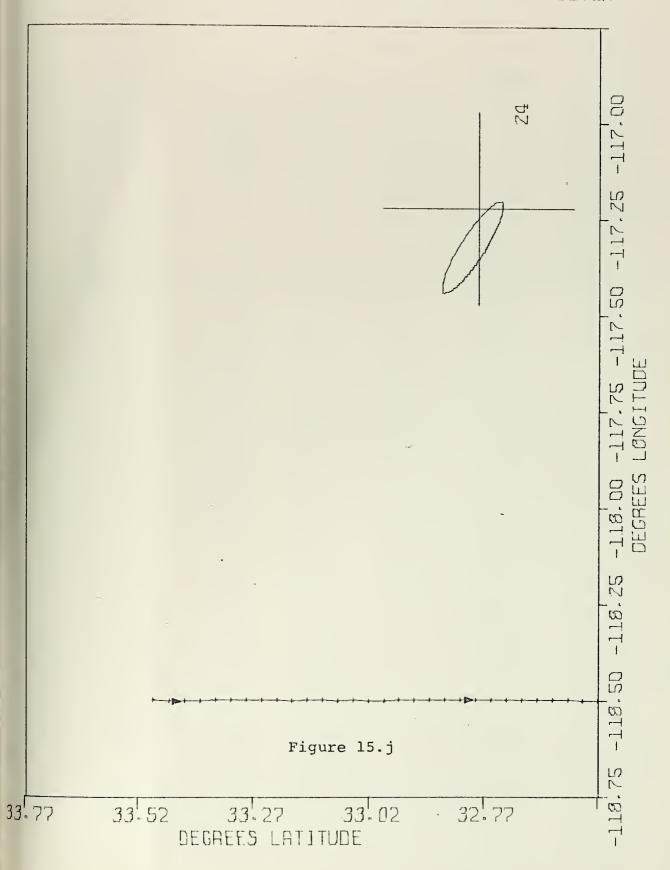






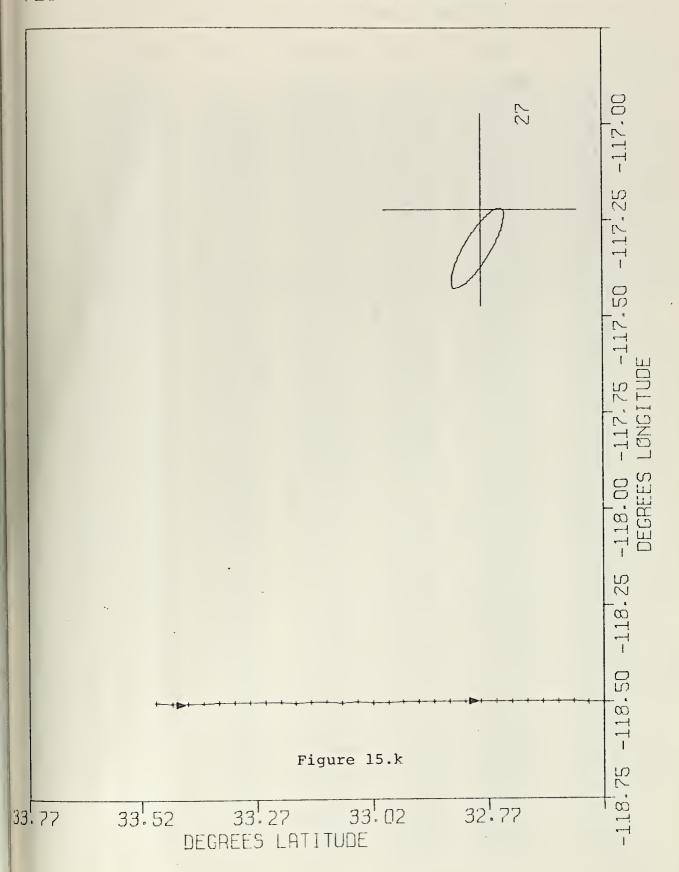


PLOT OF ERROR COVARIANCES OF EXTENDED KALMAN FILTER





# PLOT OF ERROR COVARIANCES OF EXTENDED KALMAN FILTER





## NAVIGATION DATA FILTER PARAMETERS

K	SG1	SG2	SP11	SP12	SP22	т
1234567890123467890123467890123467890123467890123467890123467890123467890123467890123467890123467890123467890123467890124567890124567890124567890124567890124567890124567890124567890124567890100000000000000000000000000000000000	0.0028816410993554830000000000000000000000000000000000	0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0	1.000000000000000000000000000000000000	0.0 19.4055759996626555999662245599000000000000000000000000000000000	1.00001 0.00038 0.000339 0.000660 0.0000339 0.000043 0.0000143 0.0000140 0.000013 0.0000140 0.0000110	91643416417611335628598115527903236402005983752227537435436422312 0456417777751898969085099960971338860984937157619899617961627719990 09946494913111111111111111111111111111111



# NAVIGATION DATA FILTER PARAMETERS

K	VELND	ELAT	SLATD	VELED	ELON	SLOND
123456789001234567890012345678900123456789001234567890012345678900123456789000123456788900012346789000000000000000000000000000000000000	-0.00096 -0.00097 -0.00095 -0.00094 -0.00095 -0.00095 -0.00095 -0.00095 -0.00097 -0.00096 -0.00066 -0.00064 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069 -0.00069	0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0 0.0	22289912170057777299562002248421524605633999133310499706000339004253889008732913310497060003390442538890087333133333333333333333333333333333333	0.00103 0.00103 0.00103 0.00103 0.001092 0.00103 0.00103 0.000199 0.0001999 0.0001999 0.0001999 0.0001999 0.000103 0.000123 0.001123 0.001123 0.001123 0.001123 0.001121	0.00 0.0033915 0.00339356 0.00339356 0.0004185 0.0004185 0.000161811 0.0001571205 0.00016171205 0.00016171205 0.0001677311667 0.0001677311667 0.0001677311667 0.00016751100	-120.8856892 -120.8856892 -120.8856892 -120.881730 -120.8776324 -120.776324 -120.776324 -120.776324 -120.77337465 -120.7337465 -120.703888 -120.703888 -120.703888 -120.703888 -120.668864 -120.668864 -120.668864 -120.668864 -120.668864 -120.668864 -120.667997 -120.5563115 -120.668864 -120.6697641 -120.6551562 -120.6480487 -120.449954 -120.449954 -120.449954 -120.449954 -120.449957 -120.339578 -120.338893 -120.338893 -120.338893 -120.338893 -120.338893 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578 -120.339578



## LOB NUMBER ASSIGNED TO TARGET I

	1	2	3	4	5	6	7	8	9	10	1.1	12	13	14	15	16	17	18		
1 2 3 4	5	18	29	34	7 35 41 25 38								33	37	40	42	48	53	54	59
5	12	17 22	47	26 52	3 8 5 6	45 62	51	55	60											



## LOB NUMBER ASSIGNED TO TARGET I

9 10 11 12 13 14 15 16 17 18 43 41 49 56 35 34 46 45 52 10 14 15 19 28 32 33 37 40 42 48 53 54 59 50 61 65 18 13 17 22 39 30 29 16 38 47 57 1123243 2162 55 60 26 27 36 58 63 64





I	J	К	T(K)	THTD(K)	TDTD(K)	E(K)	GATE(K)
111111111111111111111111111111111111111	0234567890112345678900	124679045982370283445555	0.0 19.485 34.179 19.573 14.742 24.466 9.744 24.439 9.869 73.5869 734.573 9.831 24.613 9.844 9.844 39.444 39.444 39.444 39.444 39.444	59.2273 57.74254 55.152216 55.192216 51.74072 49.875205 51.74054 47.875204 47.875204 47.875204 42.57524 37.5334 35.1283 28.1404 27.7538 28.1403 28.1403 28.1403 28.1403 28.1403	-0.0479 -0.0759 -0.0549 -0.0532 -0.0599 -0.0615 -0.0615 -0.0613 -0.06632 -0.06659 -0.06662 -0.06661 -0.0671 -0.0699 -0.06998 -0.0473	-0.5454 1.0632 0.1573 -0.8703 0.1281 -0.5299 0.6015 -0.71289 -0.82399 0.1300 -0.2954 0.1419 -0.1759 -0.2278 -0.6683 -0.1452	974 · 2915 20 · 4241 10 · 3016 8 · 6128 8 · 5028 7 · 6709 7 · 7350 7 · 3770 8 · 4975 8 · 01957 7 · 3704 7 · 3704 7 · 0729 7 · 7052 7 · 7581 7 · 1742 7 · 4181
112222222222333334	234567890234	233440155894 123	156.824 69.009 59.161 68.851 9.783 34.461 83.767 24.765 0.0 102.837 88.724 49.304 54.138	62.3831 54.0991 51.0311 48.0350 46.0922 45.30553 38.9379 37.8015 76.3184 73.9504 70.5609 69.2111	-0.0528 -0.0489 -0.0497 -0.0396 -0.0418 -0.0487 -0.0484 -0.0484 -0.0423 -0.0230 -0.0320 -0.0304	-0.8639 0.7317 -0.1518 2.2497 -0.7265 -2.1226 -0.0577 0.1693 1.9865 -1.6330 0.3671	7841.2148 13.0578 10.4168 10.2537 7.9461 7.8894 9.3306 8.0812 5141.8398 14.3243 9.8099
4 4 4	5023450	41 8 13 16 46 49	0.0 39.034 19.638 251.067 19.688	67.6206 89.8662 87.7937 86.8632 70.9110 69.7662 88.4148	-0.0300 -0.0351 -0.0531 -0.0508 -0.0623 -0.0615 -0.0363	0.0988 -0.7037 0.1565 -3.2701 0.1441	9.2293 1951.7271 11.2785 40.3285 9.0839
455555556	502345670	17 38 45 51 56 12	34.293 187.198 54.068 34.479 34.468 34.529 0.0	86.4644 76.1742 72.2144 69.2354 66.8928 63.3594 52.8601	-0.0569 -0.0551 -0.0635 -0.0708 -0.0699 -0.0790 -0.0490	-0.7063 0.3616 -1.3999 -1.3831 0.1874 -2.3061	1714.6934 50.8895 10.9320 9.1513 8.6534 8.3620
	23450	22 47 52 56 31	78.589 201.919 34.488 34.495 0.0	44.0398 34.5715 32.3674 28.9272 48.3618	-0.1122 -0.0612 -0.0621 -0.0762 -0.0478	-4.9659 13.8409 -0.1496 -2.3011	3929.4629 27.7451 9.6489 9.0884
7 7 0	230	39 57	68.884 132.894 0.0	44.3147 37.7173 37.0618	-0.0588 -0.0519 -0.0497	-0.7557 1.3159	3444.1973 21.2004
66667779999999	50230234567	23 25 27 36 58 64	9.888 24.660 73.959 172.278 34.466 9.906	36.0716 36.3181 33.0763 24.4576 20.1427 18.3441	-0.1001 -0.0111 -0.0364 -0.0463 -0.0684 -0.0790	-0.4992 2.8634 -2.5942 -2.5979 -4.6048 -2.7139	494.4937 26.4133 23.4523 19.5199 9.3756 7.8310
1 0 1 0	0 2	21 26	0.0 34.547	36.2854 35.1806	-0.0497 -0.0320	0.6114	1727.3518



TARGET NUMBER 1

## FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 1

К	FREQ	PRF	PW	THETAD	THTD	SLAD	SLOD
246790459823702834 11112333344455	2872.0 2872.0 2872.0 2872.0 2872.0	322.0 320.0 318.0 320.0 319.0 321.0 320.0 320.0 319.0 320.0 320.0 320.0 320.0 320.0	2.40 2.40 2.50 2.50 2.50 2.50 2.50 2.50 2.50 2.5	59.273 57.7483 556.2175 56.2000 52.58160 59.184160 59.183028 47.334233 42.5976 39.57243 39.57243 39.57243 39.57243 39.57243 39.572407 37.6510 37.6510 37.6510 37.6510 37.6510 37.6510 37.6510	59.273 57.485 56.1541 56.1542 55.1540 55.9216 57.477 50.4774 47.6678 40.57249 42.753449 37.58442 37.58442 37.58442 37.784 31.0143 31.0143 27.7081	33.358 33.358 33.3849 33.28698 33.22149 33.22149 33.1284 33.12844 33.10748 33.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749 34.10749	-120.8981 -120.8769 -120.8344 -120.8093 -120.7916 -120.7605 -120.7176 -120.7058 -120.6743 -120.5652 -120.4985 -120.4985 -120.4081 -120.3945 -120.3054 -120.2381 -120.2381 -120.1793

SMOOTHED INITIAL BEARING ANGLE = 59.22725 N
FILTERED FINAL BEARING ANGLE = 24.70815 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 33.98700 N

EMITTER LONGITUDE =-119.60393 W



## FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 2

K	FREQ	PRF	PW	THETAD	THTD	SLAD	SLOD
	2876.0 2878.0			62.3831 54.0991	62.3831 54.0991		-120.8529 -120.6545
	2878.0 2876.0			51.1855 47.9846	51.0311 48.0350		-120.5503 -120.4621
	2876.0 2876.0			46.8625 44.9780	46.0922 45.3923		-120.3590 -120.3431
-	2876.0 2876.0			41.8276 38.9141	43.0553 38.9379		-120.2898 -120.1640
65	2876.0	306.0	2.40	37.8948	37.8015	32.9261	-120.1269

SMOOTHED INITIAL BEARING ANGLE = 62.38292 N FILTERED FINAL BEARING ANGLE = 37.80147 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 34.09401 N

EMITTER LONGITUDE =-119.02849 W



## FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 3

K	FREQ	PRF	PW	THETAD	THTD	SLAD	SLOD
18 29 34	2846.0 2846.0 2846.0 2846.0	370.0 370.0 370.0	2.30 2.40 2.40	76.3184 73.9504 70.2744 69.3484	76.3184 73.9504 70.5609 69.2111 67.6206	33.1857 33.1215 33.0872	-120.8101 -120.6813 -120.5511 -120.4769 -120.3957

SMOOTHED INITIAL BEARING ANGLE = 76.31851 N FILTERED FINAL BEARING ANGLE = 67.62057 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 33.78937 N

EMITTER LONGITUDE =-118.17609 W



## FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 4

K	FREQ	PRF	PW	THETAD	THTD	SLAD	SLOD
13 16 46	5556.0 5556.0 5554.0 5554.0	640.0 320.0 640.0	1.10 1.00 0.90	89.8662 87.7937 86.9075 70.8386	89.8662 87.7937 86.8632 70.9110	33.2177 33.1988 33.0182	-120.7721 -120.7229 -120.6985 -120.3263 -120.2964

SMOOTHED INITIAL BEARING ANGLE = 89.86671 N FILTERED FINAL BEARING ANGLE = 69.76620 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 33.25092 N

EMITTER LONGITUDE =-119.49136 W



#### FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 5

K	FREQ	PRF	PW	THETAD	THTD	SLAD	SLOD
17 38 45 51 55	5506.0 5504.0 5504.0 5504.0 5504.0 5504.0	640.0 640.0 640.0 642.0 640.0	0.60 0.60 0.70 0.60 0.60	88.4148 86.4644 76.1792 71.7927 68.6409 66.9829 62.1721	88.4148 86.4644 76.1742 72.2144 69.2354 66.8928 63.3594	33.1942 33.0597 33.0218 32.9989 32.9749	-120.7356 -120.6923 -120.4169 -120.3343 -120.2809 -120.2294 -120.1774

SMOOTHED INITIAL BEARING ANGLE = 88.41481 N FILTERED FINAL BEARING ANGLE = 63.35942 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 33.25052 N

EMITTER LONGITUDE =-119.45798 W



## FILTERED AND SMCOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 6

К	FREQ	PRF	PW	THETAD	THTD	SLAD	SLOD
22 47 52	2876.0 2876.0 2876.0 2874.0 2876.0	359.0 359.0 359.0	2.20 2.40 2.40	52.8601 44.0398 35.2188 32.3096 27.9243	52.8601 44.0398 34.5715 32.3674 28.9272	33.1556 33.0157 32.9926	-120.7302 -120.6249 -120.3198 -120.2671 -120.2162

SMOOTHED INITIAL BEARING ANGLE = 52.85960 N FILTERED FINAL BEARING ANGLE = 28.92722 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 33.95410 N

EMITTER LONGITUDE =-119.55852 W



## FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 7

К	FREQ	PRF	PW	THETAD	THTD	SLAD	SLOD
39	9268.0 9268.0 9270.0	499.0	0.20	48.3618 44.3147 37.8228	48.3618 44.3147 37.7173	33.0578	-120.5182 -120.4130 -120.2116

SMOOTHED INITIAL BEARING ANGLE = 48.36174 N FILTERED FINAL BEARING ANGLE = 37.71735 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 34.12042 N

EMITTER LONGITUDE =-119.12958 W



FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 8

K FREQ PRF PW THETAD THTD SLAD SLOD 24 2876.0 305.0 2.40 19.1616 33.1525 -120.6167

SINGLE LINE BEARING



## FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 9

К	FREQ	PRF	PW	THETAD	THTD	SLAD	SLOD
25 27 36 58	5556.0 5556.0 5554.0 5554.0 5556.0	320.0 640.0 640.0 640.0	1.10 1.10 1.10 1.00	37.0618 36.0715 36.4658 32.9065 24.2122 18.2568	37.0618 36.0716 36.3181 33.0763 24.4576 20.1427	33.1482 33.1312 33.0797 32.9610 32.9362	-120.6227 -120.6078 -120.5719 -120.4598 -120.1989 -120.1472 -120.1331

SMOOTHED INITIAL BEARING ANGLE = 37.06157 N FILTERED FINAL BEARING ANGLE = 18.34413 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 34.27957 N

EMITTER LONGITUDE =-119.59087 W



FILTERED AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 10

K FREQ PRF PW THETAD THTD SLAD SLOD 21 5504.0 640.0 0.60 36.2854 36.2854 33.1652 -120.6456 26 5504.0 640.0 0.60 35.1807 35.1806 33.1417 -120.5935

SMOOTHED INITIAL BEARING ANGLE = 36.28545 N FILTERED FINAL BEARING ANGLE = 35.18065 W

VECTOR METHOD SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 35.15515 N

EMITTER LONGITUDE =-118.84724 W



## TARGET NUMBER 11

FILTEREC AND SMOOTHED EMITTER DATA CORRELATED TO TARGET NUMBER 11

K FREO PRF PW THETAD THTD SLAD SLOD 62 2874.0 359.0 2.40 59.5566 32.9406 -120.1563

SINGLE LINE BEARING



MASTER CONTROL PROGRAM DESCRIPTION OF VARIABLES IN THE COMMON BLOCK ACLAD ACLAMD ACLAR ACLOD ACLOMD AIRCRAFT LATITUDE READ FORM DATA ACLOR ALT BRNG AIRCRAFT AIRCRAFT LONGITUDE READ FROM DATA ALTITUDE OF BEARING ANGLE (TRUE OR RELATIVE, DEPENDING ON DATA SOURCE)
KALMAN FILTER ERROR —ERM IN ANGLE FILTER SIGNAL FREQUENCY IN MEGACYCLES BRNGD FREQ G1 G2 FILTER GAINS OR DF BEARING KALMAN GATE TEST FOR CORRELATION HDG **HDGD** AIRCRAFT HEADING (TRUF) MODEN MODET MODE OF DATA COLLECTOR OPERATION BEARINGS CORRELATED NST P11 P12 P22 PITCH DF NUMBER OF TO A TARGET ERROR COVARIANCE TERMS LONGITUDINAL ATTITUDE SIGNAL PULSE REPITION SIGNAL PULSE REPITION TERMS OF AIRCRAFT FREQUENCY PRF PRF SIGNAL SIGNAL SIGNAL PULSE PULSE PULSE FREQUENC PW WIDTH TRANSVER SE ROLL ATTITUDE OF AIRCRAFT SLA SLAD SLOD SLOD FILTERED AND SMOOTHED AIRCRAFT LATITUDE FILTERED AND SMOOTHED LONGITUDE TIME BETWEEN DATA SETS (SECONDS) TIME OF NAVIGATIONAL FIXES TIME OF TAKING OF BEARING OF AIRCRAFT TIME TIMEN DE BEARING ANGLE RATE FILTERED OF BEARING ANGLE SMOOTHED FIRST OF BEARING TOTO THTD THTD1 THETA THETAD-TLAD -RING ANGLE FROM DATA (TRUE) LATITUDE FROM ANGLE FILTERING LONGITUDE FROM ANGLE FILTERING DF BEARING TARGET LAT TARGET LON TLOD TARGET LONGITUDE FROM ANGLE FILTERING
AIRCRAFT SPEED OVER THE GROUND
AIRCRAFT VELOCITY IN EAST DIRECTION (NEGATIVE
DENOTES AIRCRAFT HEADING WEST)
AIRCRAFT VELOCITY IN NORTH DIRECTION (NEGATIVE
NUMBER INDICATES THAT THE AIRCRAFT IS HEADING
TARGET LONGITUDE FROM EXTENDED KALMAN FILTER
TARGET LATITUDE FROM EXTENDED KALMAN FILTER
COVARIANCE OF ERROR OF SMOOTHED FIRST DE VEL VELE VELN (NEGATIVE NUMBER IND TARGET LON TARGET LAT COVARIANCE BEARING NUMBER OF XTD1 YTD1 D11 OF DF BEARINGS FILTERED IN ANGLE FILTE PROCESSING SWITCHED TO EXTENDED KALMAN **JST** ANGLE FILTER BEFORE VARIABLES COMMON THROUGHOUT PROGRAM MATRIX MATRIX CONES RELATING DATA SETS TO TARGETS
OF LATITUDE OF INTERSECTION O
ASSOCIATED WITH EACH BEARING
OFLONGITUDE OF INTERSECTION O **JSET** PTLAT ERROR MATRIX CONES PTLON ERROR EACH DE BEARING CORRELATED TO A FILTERED BEFORE DEXTENDED KALMA ASSOCIATED WITH R OF DF BEARINGS ( R OF DF BEARINGS SSING SWITCHED TO NUMBER NUMBER NSTA JSTA TARGET PROCESSING KALMAN FILTER



```
C
           COMMON ACLAD(100).ACLAMD(100),ACLAR(100),ACLOD(100),
1ACLOND(100).ACLOR(100).ALT(100).BRNG(100),BRNGD(100),
2E(100).FRED(100).GI(100).G2(100).GATE(100).HDG(100),
3HDGD(100).MCDEN(100).MGDET(100).NST(100).PII(100),
4P12(100).P22(100).PITCH(100).PRF(100).PW(100),
5RGLL(100).SLA(100).SLAD(100).SLO(100).SLOD(100),
5T(100).TIMEN(100).TIMET(100).TDTD(100).THTD(100).
6THTD1(100).THETA(100).THETAD(100).TLAD(100).TLOD(100),
7VEL(100).VELE(100).VELN(100).XTD(100).YTD(100),
           8D11(100), JST(100)
DIMENSION JSET(15,50), PTLAT(4,15,50), PTLON(4,15,50),
           DIMENSION JSET(15.50), PILAT(4,15,50), PILON(
1X3(101), Y3(101)

DATA IX/531/.STDEV/1./, STDEVN/.5/, AMEAN/O./
DATA TEST/.7/
DATA JDIM/2/
DATA SIGMA/1./
DATA RNAV/.25/
CATA RXTEND/.05/
             KFLAG=1
                        EXTEST /3.0/
RCUT.TSTCUT.JDIM /144...5,15/
READ(NUM)
             DATA
             DATA
             CALL
             CALL NAV (NUM, RNAV, TEST)
CALL GEORGE (NUM, NTAR, RCUT, RXT END, EXTEST, TSTCUT, SIGMA,
           1JSET.PTLAT,PTLON,JDIM)
CCC
         SUBROUTINE TO COMPUTE FINAL EMITTER TARGET JSET DATA
             WRITE(6.66)
WRITE(6.202)
WRITE(6.203)(K,K=1.18)
DO 30 I=1.NTAR
NSTA=NST(I)
             WRITE(6, 204) I, (JSET(I, J), J=1, NSTA)
CONTINUE
  30
             K2 = 0
             WRITE(6,55)
WRITE(6,53)
DO 32 I=1.NTAR
NSTA=NST(I)
             IF(NSTA.EQ.1) GO TO 32
KI=JSET(I,1)
             WRITE(6.64) I.K2.KI,P11(KI),P12(KI),G1(KI),G2(KI)
              JSTA=JST(I)
             DO 31 J=2.JSTA
K=JSET(I.J)
              WRITE (6,64) I, J, K, P11 (K), P12 (K), G1 (K), G2 (K)
  31
32
             CONT INUE
             CONTINUE
WRITE (6.55)
WRITE (6.54)
             DO 35 I=1.NTAR
             NSTA=NST(I)
             IF(NSTA.EQ.1) GO TO 35
KI=JSET(I.1)
WRITE(6.65)I.K2.KI.T(KI).THTD(KI).TDTD(KI)
              JSTA=JST(I)
             DO 34 J=2.JSTA
K=JSET(I.J)
WRITE(6.65) I.J.K.T(K).THTD(K).TDTD(K).E(K).GATE(K)
CONTINUE
CONTINUE
             DO 37 I=1.NTAR
WRITE(6.59)I
             NSTA=NST(I)
WRITE(6.52)I
              IF(NSTA.FQ.1) GO TO 110
JSTA=JST(1)
             DO 45 J=1.JSTA
K=JSET(1.J)
```



```
WRITE(6,51)(K, FREQ(K), PRF(K), PW(K), THETAD(K), THTD(K),
                        1SLAD(K), SLOD(K))
                             CONTINUE
IF(ABS(TLAD(I)).LE.1E-6) GO TO 46
KF=JSET(I,JSTA)
WRITE(6,68)THTD1(KF).THTD(KF)
 45
                             WRITE(6,70) TLAD(I),TLOD(I)
IF(NSTA.EQ.JSTA) GO TO 37
KF=JSET(I.NSTA)
WRITE(6,71) YTD(KF),XTD(KF)
                              GO TO
                              WRITE(6,205)
46
                             GD TD 37
K=JSET(I.1)
WRITE(6.67)K.FREQ(K),PRF(K),PW(K),THETAD(K),
110
                       1 SLAD(K), SLOD(K)
WRITE(6,200)
CONTINUE
                    FORMAT (20X, 13, F7.1, F6.1, F5.2, 4F10.4)
FORMAT (20X, 'FILTERED AND SMOOTHED EMITTER DATA',
1' CORRELATED TO TARGET NUMBER ', 12, ///, 22X, 'K', 2X, 2'FREO', 3X, 'PRF', 3X, 'PW', 4X, 'THETAD', 5X, 'THTD',
36X, 'SLAD', 6X, 'SLOD', /)
FORMAT (23X, 'I', 3X, 'J', 3X, 'K', 6X, 'P11(K)', 8X,
1'P12(K)', 6X, 'G1(K)', 5X, 'G2(K)', /)
FORMAT (//, 23X, 'I', 3X, 'J', 3X, 'K', 3X, 'T(K)', 4X,
1'THTD(K)', 3X, 'TDTD(K)', 3X, 'E(K)', 5X, 'GATE(K)', /)
FORMAT ('1', 29X, 'KALMAN FILTER PARAMETERS FOR ANGLE',
1' FILTER', //)
FORMAT (20X, 314, F9.3, F10.4, 2F10.4)
FORMAT (20X, 314, F9.3, F10.4, 2F9.4, F11.4)
FORMAT (20X, 314, F9.3, F10.4, 2F9.4, F11.4)
FORMAT (20X, 314, F9.3, F10.4, 2F9.4, F11.4)
FORMAT ('1', 24X, 'FINAL JSET DATA', //)
FORMAT (20X, 314, F9.3, F10.4, 2F9.4, F11.4)
FORMAT ('1', 22X, 'SMOOTHED INITIAL BEARING ANGLE',
2' = ', F10.5, 'W', //, 35X, 'FINAL JSET DATA', //)
FORMAT ('/, 30X, 'YETTERED FINAL BEARING ANGLE',
2' = ', F10.5, 'W', //, 36X, 'EMITTER LATITUDE = ',
2F10.5, 'N', //, 35X, 'EMITTER LONGITUDE = ', F10.5, 'W')
FORMAT ('/, 35X, 'EXTENDED KALMAN FILTER SOLUTION OF',
1' EMITTER LOCATION', //, 36X, 'EMITTER LATITUDE = ',
2F10.5, 'N', //, 35X, 'EMITTER LONGITUDE = ', F10.5, 'W')
FORMAT ('/, 40X, 'SINGLE LINE BEARING DO NOT CROSS',
1' **', 30X, '* THEREFORE NO SOLUTION IS POSSIBLE.',
2'*')
FORMAT ('//, 30X, '* THEREFORE NO SOLUTION IS POSSIBLE.',
2'*')
37
51
52
53
54
55
59
64
65
66
67
68
70
71
200
202
203
204
 205
                        2 1 % 1 )
                              STOP
                     · END
```



C

```
SUBROUTINE NAV (NUM. RNAV. TEST)
SUBROUTINE TO FILTER AND SMOOTH NOISY AIRCRAFT
NAVIGATION
                       DATA
LIST OF VARIABLES
A11
A12
A21
A22
ELAT
                 SMOOTHING FILTER GAINS
LATITUDE FILTER ERROR
ELAD1
                 LATITUDE SMOOTHING FILTER ERROR TERM LONGITUDE FILTER ERROR TERM
ELAD2
ELON1
ELON2
LEG1
                 LONGITUDE
                                     SMOOTHING FILTER CORRESPONDING TO
                                                                         ERROR
FIRST
                                                                                             FIX
                                                                                                      ΙN
                 LEG
LAST
WORK
                          NAV FIX IN
VECTOR FOR
VECTOR FOR
ARGUMENT
LEG2
LW
                                                  MINV
MW
                 WORK
                                                  MINV
PIN
PIN11
PIN12
PIN22
                 MINV
                 INVERSE OF ERROR COVARIANCE
Q11
Q12
Q22
SLAD
SLATD
SLOND
                 VARIANCE
AIRCRAFT
                                   OF
                                          SYSTEM
                                                        NOISE
                                   LATITUDE
            -
-
                 PREDICTED AIRCRAFT
                                                                          ESTIMATE
ESTIMATE
                                                        LATITUDE
                 PREDICTED AIRCRAFT
                                                        LONGITUDE
SLUND -
SLOD -
SG1 -
SG2 -
SP11
SP12
SP22 -
SPKK11
SPKK12
SPKK22-
                 AIRCRAFT LONGITUDE
                KALMAN FILTER GAIN
                 ERROR COVARIANCE TERM - (K+1/K)
                ERROR COVARIANCE TERM - (KETIME INCREMENT DIVIDED BY WAIRCRAFT VELOCITY EAST IN A STRUCK AFT VELOCITY AIRCRAFT VELOCITY AIRCRAFT VELOCITY NORTH IN SMOOTHED AIRCRAFT VELOCITY AIRCRAFT VELOCITY AIRCRAFT VELOCITY AIRCRAFT VELOCITY NORTH IN NUMBER OF LEGS VARIANCE OF MEASUREMENT NO
                                                                KKK)
BY W
IN DEG
VELED -
VELEDS -
VELEM -
                                                                       DEGREES
                                                                       EAST
KNOTS
VELND -
VELNDS-
VELNM -
NLEG -
                                                                         DEGREES
                                                                         NORTH
KNOTS
                 VARIANCE (
RNAV
                                                                   NOISE
```

## SUBROUTINE NAV (NUM, RNAV, TEST)

COMMON ACLAD(100), ACLAMD(100), ACLAR(100), ACLOD(100), 1ACLOMD(100), ACLGR(100), ALT(100), BRNG(100), BRNGD(100), 2E(100), FREQ(100), GI(100), GZ(100), GATE(100), HDG(100), 3HDGD(100), MODEN(100), MODET(100), NST(100), P1I(100), 4P12(100), P22(100), PITCH(100), PRF(100), PW(100), 5RGLL(100), SLA(100), SLADSM(100), SLO(100), SLODSM(100), 5T(100), TIMEN(100), TIMET(100), TDTD(100), THTD(100), 6THTD1(100), THETA(100), THETAD(100), TLAD(100), TLOD(100), 7VEL(100), VELE(1CO), VELN(100), XTD(100), YTD(100), 8D11(100), JST(100)

DIMENSION A11(100), A12(100), A21(100), A22(100), 1ELON1(100), 2ELON2(100), LEGI(50), LEG2(50), LW(2), MW(2), PIN(2,2),



```
3PIN11(100), PIN12(100), PIN22(100), Q11(100), Q12(100), 4022(100), SLAD(100), SLATD(100), SLOND(100), SLOD(100), 5, SG1(100), SG2(100), SP11(100), SP12(100), SP22(100), 6SPKK11(100), SPKK12(100), SPKK22(100), TT(100), VELED(100), VELEDS(100), VELDS(100), 
C
                             DATA PIRAD/57.29578/
00000
               PROGRAM TO FILTER NOISY MEASURED LATITUDE AND LONGITUDE
               DIVIDE THE FLIGHT TRACK INTO LEGS FOR FILTERING
                             LEG1(1)=1
                           L = L + 1
    1
                             CONTINUE
                             LEG2 (L) = NUM
                            NLEG=L
00000
                   START OF KALMAN FILTER PREDICTION PROBLEM
               INITALIZE KALMAN FILTER
                            DO 100 L=1.NLEG
LEG2L=LEG2(L)
                          LEG1L=LEG1(L)

KI=LEG1L

ELON(KI)=0.0

ELAT(KI)=0.0

SG1(KI)=0.0

SG2(KI)=0.0

SPKK11(KI)=1.

SPKK12(KI)=1.

SPHX22(KI)=1.

SP12(KI)=0.

SP22(KI)=1.

SP12(KI)=0.

SP22(KI)=1.

SLAD(KI)=ACLAD(KI)

SLATD(KI)=ACLAD(KI)

VELND(KI)=ACLAD(KI)

VELND(KI)=ACLAD(KI)

VELND(KI)=ACLOD(KI)

VELNM(KI)=VELND(KI)=ACLOD(KI)

VELNM(KI)=VELND(KI)=ACLOD(KI)

VELNM(KI)=ACLOD(KI)
                             LEGIL=LEGI(L)
                             SLOD(KI) = ACLOD(KI)

VELED(KI) = ((ACLOD(KI+1)-ACLOD(KI))/T(KI+1))*COS(SLA(KI

VELEM(KI) = VELED(KI)*216000.
000
               START OF KALMAN FILTER RECURSION EQUATIONS
                             LG1P1=LEG1L+1
CO 6 K=LG1P1,LEG2L
                             KK = K - 1
                             TT(K)=T(K)/1000.0
Q11(K)=TT(K)性4/4.0
Q12(K)=TT(K)性3/2.0
Q22(K)=TT(K)性2
               COMPUTE KALMAN FILTER GAINS
                              SP11(K)=SPKK11(KK)+(2.0字SPKK12(KK)+SPKK22(KK)本T(K))字
                         1T(K)+011(K)
SP12(K)=SPKK12(KK)+SPKK22(KK)*T(K)+012(K)
                              SP22(K) = SPKK22(KK) + 022(K)
```



```
SG1(K)=SP11(K)/(SP11(K)+RNAV*T(K)/3600.)

SG2(K)=SP12(K)/(SP11(K)+RNAV*T(K)/3600.)

SPKK11(K)=SP11(K)*(1.0-SG1(K))

SPKK12(K)=SP12(K)*(1.0-SG1(K))

SPKK22(K)=SP22(K)-SP12(K)*SG2(K)
CCC
     FILTER LATTITUDE
          IFLAG=1
SLATD(K)=SLAD(KK)+VELND(KK)*T(K)
  700
          ELAT(K) = ACLAD(K) - SLATD(K)
CCC
     TEST FOR EXCESS SYSTEM NOISE
          IF(ABS(ELAT(K)).LT.TEST) GO TO 800
     THE DATA POINT ACLAD(K) IS INVALID, SO IT IS DISCARDED
          IF(IFLAG.EQ.2) GO TO 800 ACLAD(KK)=SLATD(KK)
          ELAT(KK)=0.

SLAD(KK)=SLATD(KK)

VELND(KK)=VELND(KK-1)

VELNM(KK)=VELNM(KK-1)

SLA(KK)=SLAD(KK)/PIRAD
          IFLAG=2
GO TO 700
IFLAG=1
 800
     THE DATA POINT ACLAD(K) IS VALID, SO PROCEED
          SLAD(K) = SLATD(K) + SG1(K) * ELAT(K) 
VELND(K) = VELND(KK) + SG2(K) * ELAT(K) 
VELNM(K) = VELND(K) * 216000 •
          SLA(K) = SLAD(K) / PIRAD
CCC
     FILTER LONGITUDE
          SLOND(K)=SLOD(KK)+VELED(KK)與T(K)
ELON(K)=(ACLOD(K)-SLOND(K))率COS(SLA(K))
 900
CCC
     TEST FOR EXCESS SYSTEM NOISE
          IF(ABS(ELON(K)).LT.TEST) GO TO 1000
CCC
     THE DATA POINT ACLOD(K) IS INVALID, SO IT IS DISCARDED
          IF(IFLAG.EQ.2) GO TO 1000
ACLOD(KK)=SLOND(KK)
          ELON(KK)=0.
SLOD(KK)=SLOND(KK)
VELED(KK)=VELED(KK-1)
VELEM(KK)=VELEM(KK-1)
          IFLAG= 2
          GO TO 900
          IFLAG=1
  1000
     THE DATA POINT ACLOD(K) IS VALID, SO PROCEED
          SLOD(K)=SLOND(K)+SG1(K)*ELON(K)
VELED(K)=(VELED(KK)+SG2(K)*ELON(K))*COS(SLA(K))
VELEM(K)=VELED(K)*216000.
 100
          CONTINUE
     START OF FIXED INTERVAL SMOOTHING EQUATIONS
          DO 13 L=1.NLEG
LEG2L=LEG2(L)
          LEGIL=LEGI(L)
N=LEG2L-LEGIL
IF(N.LE.3) GO
                                  TO
C
```



```
C
       INITALIZE SMOOTHING FILTER
             SLADSM(LEG2L) = SLAD(LEG2L)
VELNDS(LEG2L) = VELND(LEG2L)
SLODSM(LEG2L) = SLOD(LEG2L)
VELEDS(LEG2L) = VELEC(LEG2L)
       COMPUTE SMOOTHING FILTER GAINS
             DO 10 I = 1 \cdot N
             K=LEG2L-I
             KK = K + 1
             PIN(1.1)=SP11(K)
PIN(1.2)=SP12(K)
PIN(2.1)=SP12(K)
PIN(2.2)=SP22(K)
CALL MINV(PIN. 2. DET. LW. MW)
PIN11(K)=PIN(1.1)
             PIN12(K)=PIN(1,2)
PIN22(K)=PIN(2,2)
           A11(K) = SPKK11(K) * PIN11(K) + SPKK12(K) * (PIN11(K) * T(K) + 1PIN12(K))
A12(K) = SPKK11(K) * PIN12(K) + SPKK12(K) * (PIN12(K) * T(K) + 1PIN22(K))
           A21(K)=SPKK12(K)*PIN11(K)+SPKK22(K)*(PIN11(K)™T(K)+1PIN12(K))
A22(K)=SPKK12(K)*PIN12(K)+SPKK22(K)*(PIN12(K)*T(K)+
           1PIN22(K))
       SMOOTH LATITUDE ESTIMATES
             ELAD1(K)=SLADSM(KK)-SLATD(KK)
ELAD2(K)=VELNDS(KK)-VELND(KK)
SLADSM(K)=SLAD(K)+A11(K)=ELAD1(K)+A12(K)*ELAD2(K)
             VELNDS(K) = VELND(K) + A21(K) * ELAD1(K) + A22(K) * ELAD2(K) VELNDS(K) = VELND(K) + (A21(K) + A22(K)) * ELAD1(K)
             SLA(K)=SLADSM(K)/PIRAD
       SMOOTH LONGITUDE ESTIMATES
             SLO(K)=SLODSM(K)/PIRAD
CONTINUE
  10
             GO TO 13
0000
             ANY LEGS WITH 3 OR LESS POINTS, ASSIGN THE DATA VALUES THE FILTERED AND SMOOTHED VALUES FOR THAT LEG
       AS
             DO 12 J=LEG1L, LEG2L
SLADSM(J)=SLAD(J)
SLA(J)=SLAD(J)/PIRAD
VELNDS(J)=O.
SLODSM(J)=SLOD(J)
SLO(J)=SLOD(J)/PIRAD
VELSDS(J)=O.
  11
              VELEDS(J)=0.
  12
             CONTINUE
WRITE(6.52)
WRITE(6.53)
  13
          WRITE(6.50) (K, SG1(K), SG2(K), S. 21)

1T(K), K=1, NUM)

WRITE(6.52)

WRITE(6.54)

WRITE(6.51) (K, VELND(K), ELAT(K), SLATD(K), VELED(K),

1ELON(K), SLOND(K), K=1, NUM)

FORMAT( 24X, I4, 5F9.5, F9.2)

FORMAT( 24X, I4, 2F9.5, F11.5, 2F9.5, F11.5)

FORMAT('1', /////, 38X, 'NAVIGATION DATA FILTER',

1' PARAMETERS', //)

FORMAT(27X, 'K', 4X, 'SG1', 6X, 'SG2', 6X, 'SP11',
             WRITE(6.50)
                                        (K, SG1(K), SG2(K), SP11(K), SP12(K), SP22(K),
  50
51
  52
                                        'K',4X,'SG1', 6X, 'SG2', 6X, 'SP11', 5X,
  53
```



1'SP12'. 5X, 'SP22'. 7X, 'T'.//)
54 FORMAT( 27X, 'K'.3X,'VELND', 5X, 'ELAT', 7X, 'SLATD', 14X, 'VELED'. 5X, 'ELON'. 5X, 'SLOND', //)
RETURN
END



```
SUBROUTINE GEORGE (NUM, NTAR, RCUT, RXTEND, EXTEST, TSTCUT, 1SIGMA, JSET, PTLAT, PTLON, JDIM)
              THIS FORTRAN PROGRAM IS DESIGNED TO ANALYZE AND SORT ELECTRONIC EMITTER PARAMETERS AND AIRCRAFT NAVIGATION DATA. TO FILTER EMITTER DATA USING KALMAN FILTER TECHNIQUES TO MINIMIZE BEARING ANGLE-OF-ARRIVAL MEASUREMENT NOISE. TO SMOOTH INITIAL UNFILTERED BEARING ANGLES, AND TO PREDICT EMITTER LOCATIONS USING VECTOR
               METHODS
                      LIST OF VARIABLES NOT IN COMMON BLOCK
                     NJSET
IDID1
                                                  DATA POINTS TO BE ASSOCIATED TO A TARGET SMOOTHED BEARING RATE
                                                  SMOOTHED BEARING RATE
THETAD(K+1[K)
VARIANCE OF SYSTEM NOISE
                      TPTD
                      ŤŤ
                     Wil
Wil
                     W21
W22
                                                  SMOOTHING FILTER GAINS
X COORDINATE OF ERROR ELLIPSE
Y COORDINATE OF ERROR ELLIPSE
                     X3
Y3
                      SMOTH1
                     SMOTH2
SMOTH3
SMOTH4-
DELFRE-
                                                  INTERMEDIATE TERMS IN SMOOTHING COMPUTATIONS CARRIER FREQUENCY TEST
                                                 CARRIER FREQUENCY TEST
PULSE REPITION FREQUENCY TEST
NUMBER OF TARGETS
VARIANCE OF DF BEARING MEASUREMENT ERROR
VARIANCE OF MEASUREMENT NOISE IN EXTEND
MULTIPLIER FOR TEST TO SWITCH TO EXTEND
MULTIPLIER FOR BEARING CORRELATION TEST
MULTIPLIER TO VARY SIZE OF ERROR ELLIPSE
DEEINED BY POINTS
                     DELPRF-
                     NTAR
                     RCUT
                     RXTEND-
EXTEST-
TSTCUT-
                      SIGMA -
                  SUBROUTINE GEORGE(NUM, NTAR, RCUT, RXTEND, EXTEST, TSTCUT, 1SIGMA, JSET, PTLAT, PTLON, JDIM)
                 COMMON ACLAD(100).ACLAMD(100).ACLAR(100),ACLOD(100),
1ACLOMD(100).ACLOR(100).ALT(100).BRNG(100).BRNGD(100),
2E(100).FREQ(100).G1(100).G2(100).GATE(100).HDG(100),
3HDGD(100).MGDEN(100).MGDET(100).NST(100).P11(100),
4P12(100).P22(100).PITCH(100).PRF(100).PW(100),
5RCLL(100).SLA(100).SLAD(100).SLOD(100).SLOD(100),
5T(100).TIMEN(100).TIMET(100).TDTD(100).THTD(100),
6THTD1(100).THETA(100).THETAD(100).TLAD(100).TLOD(100),
7VEL(100).VELE(100).VELN(100).XTD(100).YTD(100),
8D11(100).JST(100)
DIMENSION JSET(JCIM.50).NJSET(100).PTLAT(4.JDIM.50).
1PTLON(4.JDIM.50).Q11(100).Q12(100).Q22(100),
2TOTD1(100).TPTD(100).TT(100).W11(100).W12(100),
3W21(100).W22(100).X3(101).Y3(101)
C
C
                      DATA PIRAD/57.29578/
              PROGRAM TO SORT EMITTER TARGET DATA AND ESTIMATE NUMBER OF DISTINCT EMITTER TARGETS
DATA IS INITIALLY SORTED BY FREQUENCY AND PRF
```

1

NF=NUM

DO 6 I=1, NUM

CUTERR=3.5 \* SORT(RCUT)
DO 1 I=1.NUM
NJSET(I)=I



```
K=NJSET(1)
    JSET(I.1)=K
NSORT=NF
    NF=0
    NJ=1
    DO 5 J=2.NSORT
KK=NJSFT(J)
DEL FRE= FREQ(K) - FREQ(KK)
    DELPRE=PRE(K)-PRE(KK)

IF(ABS(DELFRE).LE.30.0.AND.ABS(DELPRF).LE.10.) GO TO 4

IF(ABS(DELFRE).GT.30.0) GO TO 3

IF(PRE(K).EQ.0.0.OR.PRF(KK).EQ.0.0) GO TO 4

DO 2 L=2.2

RK=L*PRE(KK)
    TK=PRF(KK)/FLOAT(L)
    SK=PRF(K)
CK=ABS(RK-SK)
DK=ABS(TK-SK)
    FK=L*10.0
IF(CK.LE.FK.DR.DK.LE.FK) GO TO 4
    CONT INUE
    NF=NF+1
NJSET(NF)=KK
    GO TO
    NJ=NJ+1
JSET(I.NJ)=KK
CONTINUE
NST(I)=NJ
    IF(NF.LE.1)
                            GO TO 7
   NTAR=I+1

IF(NF.EQ.O) GC TO 8

NTAR=NTAR+1

JSET(NTAR.1)=NJSET(1)

NST(NTAR)=1
    WRITE(6.202)
WRITE(6.203)(K,K=1.18)
DO 9 I=1.NTAR
    NSTA=NST(I)
    WRITE(6, 204) I. (JSET(I.J).J=1.NSTA)
CONTINUE
PROGRAM TO FILTER NOISY MEASURED BEARING ANGLE THETAD(K) AND AIRCRAFT NAVIGATION DATA FOR EACH SUSPECTED EMITTER
TARGET OF SIMILAR FREQUENCY AND PRE
INITIALIZATION OF KALMAN FILTER EQUATION PARAMETERS
    IFLAG=1
    NTAR1=1
    NUMTAR=NTAR
DO 27 I=NTAR1.NUMTAR
    NJ=0
    NSTA2=NST(I)
NSTA=NST(I)
IF(NSTA.EQ.1) GO TO 27
KI=JSET(I.1)
    P11(KI)=10000.0
P12(KI)=0.0
P22(KI)=10000.0
    011(KI)=0.0
012(KI)=0.0
    O22(KI)=0.0
D11(KI)=1.0
W11(KI)=1.
W12(KI)=0.
   W12(KI)=0.

W22(KI)=0.

W22(KI)=1.

G2(KI)=P12(KI)/(P11(KI)+RCUT)

G1(KI)=P11(KI)/(P11(KI)+RCUT)

THTD(KI)=THETAD(KI)
    THTD1(KI)=THET AD(KI)
```



```
TDTD(KI) = VEL(KI) # SIN(BRNG(KI)) * PIRAD/600000.0
           TDTD1(KI)=TDTD(KI)
           T(KI) = 0.0
       START OF KALMAN FILTER PREDICTION PROBLEM
           DO 23 J=2.NSTA2
K=JSFT(I.J)
KK=JSET(I.J-1)
TKM1=TIMET(KK)
  11
           T(K)=TIMET(K)-TKM1
TT(K)=T(K)/1000.0
Q11(K)=TT(K)/204/4.0
Q12(K)=TT(K)/204/2.0
Q22(K)=TT(K)/202/2.0
       START OF KALMAN FILTER RECURSION EQUATIONS
         P11(K)=P11(KK)*(1.0-G1(KK))+2.0*P12(KK)*T(K)-(P12(KK)*1G1(KK)+P11(KK)*G2(KK))*T(K)+(P22(KK)-P12(KK)*G2(KK))*
         2T(K) 年本2+Q11(K)
           P12(K)=P12(KK)*(1.0-G1(KK))+(P22(KK)-P12(KK)*G2(KK))*
         1T(K)+012(K)
P22(K)=P22(KK)-P12(KK)*G2(KK)+022(K)
           G1(K)=P11(K)/(P11(K)+RCUT)
G2(K)=P12(K)/(P11(K)+RCUT)
TPTD(K)=THTD(KK)+TDTD(KK)*T(K)
E(K)=THETAD(K)-TPTD(K)
  12
           THTD(K) = TPTD(K) + G1(K) + E(K)
           TDTD(K) = TDTD(KK) + G2(K) *E(K)
00000
       CORRELATION GATING SCHEME TO ESTIMATE WHETHER FILT
BEARING ANGLE THTD(K) IS AN EMISSION FROM EMITTER
NTAR = I OR A SPURIOUS EMISSION
           GATE(K)=SQRT(P11(K)+RCUT)*TSTCUT
IF(ABS(E(K)).LT.GATE(K)) GO TO 22
IF(IFLAG.EQ.1) GO TO 14
IF(IFLAG.EQ.2) GO TO 17
  13
           GO TO 18

IF(THETAD(KK).LT.10.0.AND.THETAD(K).GT.350.0) GO
IF(THETAD(KK).GT.350.0.AND.THETAD(K).LT.10.0) GO
  14
           THETAD(K)=THETAD(K)-360.0
THETA(K)=THETA(K)-6.283186
  15
           IFLAG=2
           GO TO 12
           THETAD(K)=THETAD(K)+360.0
  16
           THETA(K)=THETA(K)+6.283186
           IFLAG=3
GO TO 12
THETAD(K)=THETAD(K)+360.0
  17
           THETA(K)=THETA(K)+6.283186
           GO TO 19
THETAD(K)=THETAD(K)-360.0
  18
           THETA(K)=THETA(K)-6.283186
           IFLAG=1
           NJ=NJ+1
JSET (NTAR+1•NJ)=K
NST(NTAR+1)=NJ
  20
THTD(K) IS A SPURIOUS BEARING LINE AND IS ASSIGNED TO A NEW TARGET.
           NST(I) = NST(I) - 1
          NSTA2=NST(I)
IF(NSTA2.LT.J) GO TO
DO 21 L=J.NSTA2
JSET(I.L)=JSET(I.L+1)
GO TO 11
                                     GO TO 24
  21
c<sup>22</sup>
           IFLAG=1
```



```
SMOOTHING EQUATIONS FOR FIRST BEARING LINE ESTIMATE
              SMOTH1=1.0-P11(KK)*(1.0-G1(KK))/RCUT

SMOTH2=SMOTH1*T(K)

SMOTH3=-P12(KK)*(1.0-G1(KK))/RCUT

SMOTH4=1.0+SMOTH3*T(K)

W11(K)=W11(KK)*SMOTH1+W12(KK)*SMOTH2

W12(K)=W11(KK)*SMOTH3+W12(KK)*SMOTH4

W21(K)=W21(KK)*SMOTH1+W22(KK)*SMOTH4

W21(K)=W21(KK)*SMOTH1+W22(KK)*SMOTH4

D11(K)=U11(KK)+W11(K)*CUT)*E(K)

THTD1(K)=THTD1(KK)+(W11(K)/RCUT)*E(K)
               TDTD1(K) = TDTD1(KK) + (W21(K)/RCUT) \approx E(K)
               IF(Pli(K).LT.(EXTEST*RCUT)) GO TO 25
    23
              CONTINUE
C
C
C
C
C
           FILTERED BEARING ANGLE THTD(K) IS CORRELATED TO EMITTER
           TARGET NTAR = I
              IF(NSTA2.NE.NSTA) NTAR=NTAR+1
IF(NSTA2.EQ.1) GO TO 27
  CCC
           PROGRAM TO COMPUTE TARGET POSITION
              KI=JSET(I.1)
KF=JSET(I.NSTA2)
CALL PREPAR (KI.KF,SLAD,SLOD,THTD1(KF),THTD(KF),
             1TLAD(I), TLOD(I))
GO TO 27
  000
         PROCESSING SWITCHES TO
                                                         EXTENDED KALMAN FILTERING
            CALL POINTS(I.J., SIG MA.RCUT, D11. HDGD.JSET, P11.SLAD, 1SLOD.THTD1.THTD.PTLAT.PTLON.JDIM.&27)
DO 26 N=1.4
X3(N)=PTLON(N.I.J)
    25
             Y3(N) = PTLAT(N, I, J)
CALL EXTEND(I, J, NSTA2, RXTEND, TSTCUT, JSET, NST, SLAD,
1SLOD, T, THETA, THETAD, THTD1, THTD, TLAD, TLOD, XTD, YTD, X3,
    26
            2Y3.JDIM)
IF(NSTA2.NE.NSTA) NTAR=NTAR+1
               CONT INUE
    27
              IF(NUMTAR.EQ.NTAR) RETURN
NTAR 1=NUMTAR+1
GO TO 10
FORMAT('1'.35X,'LOB NUMBER ASSIGNED TO TARGET I',//)
FORMAT(26X,1813,//)
FORMAT(26X,1813,//)
    202
203
     204
               FORMAT (20X, 13, 3X, 3213)
               END
```



C

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SUBROUTINE EXTEND(I, J. NSTA, RXTEND, TSTCUT, JSET, NST, 1SLAD, SLOD, T. THETA, THETAD, THTD1, THTD, TLAD, TLOD, XTD, 2YTD.X3,Y3,JDIM) SUBROUTINE TO COMPUTE EMITTER LOCATION USING EXTENDED KALMAN FILTERING TECHNIQUES. TARGET NUMBER
CUT NUMBER CORRELATED TO TARGET I
SEMI-MAJOR AXIS OF ERROR ELLIPSE
SEMI-MINOR AXIS OF ERROR ELLIPSE
ANGLE OF ROTATION OF ERROR ELLIPSE
VECTOR OF A/C LATITUDES IN DEGREES
VECTOR OF A/C LATITUDES IN RADIANS
VECTOR OF A/C LONGITUDES
VARIANCE OF MEASUREMENT NOISE
VARIANCE OF MEASUREMENT NOISE
VARIABLE USED TO VARY SIZE OF GATE
LATITUDE OF TARGET
LATITUDE OF TARGET
LATITUDE OF TARGET
LONGITUDE OF TARGET
LONGITUDE OF TARGET
LONGITUDE OF TARGET
DISTANCE FROM TARGET
TO STANCE FROM TARGET
TO STANCE FROM TARGET
RATE OF CHANGE OF DF BEARING WITH RE
TO CHANGE OF LONGITUDE OF TARGET
RATE OF CHANGE OF DF BEARING WITH RE
TO CHANGE OF LATITUDE OF TARGET
KALMAN FILTER GAIN IN X
DF BEARING FROM IN X
DF BEARING FROM IN LONGITUDE
COVARIANCE OF ERROR IN LONGITUDE
COVARIANCE OF ERROR
VARIANCE OF SYSTEM ERRORS
COVARIANCE OF SYSTEM ERRORS
COVARIANCE OF SYSTEM ERRORS LIST OF VARIABLES A ALPHA THETA SLAD SLA SLOD SLO NST EACH TARGET RXTEND TSTCUT YT YTD TLAD XT XTD TLOD AT1 AT2 DEGREES DEGREES TX DMX/DM RESPECT DMY/DM GY GX ER NSTA GATE P11 P12 P22 011 012 022 EP11 EP12 EP22 POSITION FILTERS ERROR COVARIANCE TERMS FOR LINEAR G1 G2 -XTDDOT -XTD1 -YTD1 -LINEAR FILTER GAINS
- XTD1(K+1|K)
ESTIMATE OF TARGET
ESTIMATE OF TARGET
YTD1(K+1|K)
LONGITUDE ERROR
LATITUDE ERROR LONGITUDE LATITUDE YTDDOT-EX -FY -

SUBROUTINE EXTEND(I.J.NSTA.RXTEND.TSTCUT, JSET.NST. 1 SLAD.SLOD.T.THETA.THETAD.THTD1,THTD,TLAD.TLOD,XTD. 2YTD.X3,Y3,JDIM)

DIMENSION P11(80), P12(80), P22(80), XT(80), XTD(80), YT(80), YTD(80), SLA(80), SLA(80), SLO(80), SLOD(80), THETA(80),



```
2NST(80).ER(80).TX(80),DMX(80),DMY(80),GX(80),
3GY(80).TLAD(30).TLOD(30).011(80).012(80).022(80),
4T(1).JSET(JDIM.50).TT(80).GATE(80).THETAD(1).EP11(80),
5EP12(80).EP22(80).G1(80).G2(80).XTD1(80).XTDDDT(80),
6XTDHAT(80).YTD1(30).YTDDDT(80).YTDHAT(80).EX(80),
7EY(80).THTD1(1).THTD(1).X3(4).Y3(4).Z(4).XX(101).
           8YY(101)
C
              DATA PIRAD/57.29578/
000
         EXTENDED KALMAN FILTER INITIALIZATION
             WRITE(6.102)J. I
K1=JSET(I.1)
KI=JSET(I.J)
C
           CALL PREPAR (K1.KI.SLAD, SLOD, THTD1(K1), THTD(K1), 1TLAD(I), TLOD(I))
00000000
       TLAD. TLOD IS THE CURRENT ESTIMATE OF THE TARGET
       POSITION.
                  ARE THE LOCATIONS OF THE INTERSECTIONS OF THE OF THE TWO ERROR CONES.
       X3.Y3
       EDGES
             TLA=TLAD(I)/PIRAD
DELX=(X3(1)-X3(2))*COS(TLA)
DELY=Y3(1)-Y3(2)
ALPHA1=ATAN2(DELX,DELY)
ALPHA2=ALPHA1-1.5737963
ALPHAD=ALPHA1*PIRAD
000
       ALPHAD IS THE ANGLE OF ROTATION OF THE ELLIPSE
             DELX=(TLOD(I)-X3(2))株COS(TLA)
DELY=TLAD(I)-Y3(2)
A=SORT(DELX+02+DELY+++2)
       A IS THE SEMI-MAJOR AXIS
             DO 19 K=3.4
DELX=(TLOD(I)-X3(K))*COS(TLA)
DELY=TLAD(I)-Y3(K)
BETA=ATAN2(DELX,DELY)
             GAMMA=ALPHA2-BETA
Z(K)=SQRT(DELX卒年2+DELY***2)**COS(GAMMA)
C=(ABS(Z(3))+ABS(Z(4)))/2.0
   19
CCC
          IS THE SEMI-MINOR AXIS
             WRITE(6.103) ALPHAD.A.C
CCC
       INITIALIZE THE KALMAN FILTERS
             WRITE(6.56)
YTD(KI)=TLAD(I)
             XTD(KI)=TLOD(I)

XT(KI)=XTD(KI)/PIRAD

YT(KI)=YTD(KI)/PIRAD

SLA(KI)=SLAD(KI)/PIRAD

SLO(KI)=SLOD(KI)/PIRAD

CA=COS(ALPHAI)
              SA=SIN(ALPHA1)
              SA=SIN(ALPHAI)
P11(KI)=(A常SA) 作 2+(C作CA) 作 2
P12(KI)=SA《CA》(A常将2-C培育2)
P22(KI)=(A中CA) 作 2+(C常SA) 常 2
AT1=(XT(KI)-SLO(KI)) 作 COS(YT(KI))
AT2=YT(KI)-SLA(KI)
             TX(KI)=ATAN2(AT1,AT2)
ER(KI)=THETA(KI)-TX(KI)
DM=(YTD(KI)-SLAD(KI))**2+(XTD(KI)-SLOD(KI))***2
```



```
1事(COS(YT(KI))) ##2
DMX(KI) = (YTD(KI)-SLAD(KI)) #COS(YT(KI))/DM
            DMY(KI)=-(XTD(KI)-SLOD(KI))*((YTD(KI)-SLAD(KI))*
1SIN(YT(KI))+CDS(YT(KI)))/DM
DG=Pl1(KI)*DMX(KI)**2+2.0*Pl2(KI)*DMX(KI)*DMY(KI)
            1+P22(KI)*DMY(KI)***2+RXTEND
GX(KI)=(P11(KI)**DMX(KI)+P12(KI)**DMY(KI))/DG
GY(KI)=(P12(KI)**DMX(KI)+P22(KI)**DMY(KI))/DG
              GY(KI)=(P12(KI)*DMX(KI)+P22(KI)*D

EP11(KI)=P11(KI)

EP12(KI)=P12(KI)

EP22(KI)=P22(KI)

G1(KI)=EP11(KI)/(EP11(KI)+RXTEND)

G2(KI)=D00

EX(KI)=0.0

YIDDOT(KI)=.001
              XTDDOT(KI) = .001
XTD1(KI) = XTD1(KI) + GX(KI) & ER(KI)
YTDDOT(KI) = .001
              YTD1(KI)=YTD(KI)+GY(KI)*ER(KI)
 00000
          START OF KALMAN FILTER PREDICTION PROBLEM
        COMPUTE GAINS FOR LINEARIZED FILTER
              NSTA=NST(I)
             NSIA=NSI(I)
JP1=J+1
DD 12 JJ=JP1.NSTA
K=JSET(I.JJ)
KK=JSET(I.JJ-1)
SLA(K)=SLAD(K)/PIRAD
SLO(K)=SLOD(K)/PIRAD
TT(K)=T(K)/1E3
Q11(K)=TT(K)空往4/4.0
Q12(K)=TT(K)空音3/2.0
Q22(K)=TT(K)空音
   20
              Q22(K)=TT(K)空率2
          START OF EXTENDED KALMAN FILTER RECURSION EQUATIONS
              P11(K)=P11(KK)@(1.0-GX(KK)@DMX(KK))-P12(KK)@GX(KK)@
            1DMY(KK)+011(K)
P12(K)=P12(KK)*(1.0-GX(KK)*DMX(KK))-P22(KK)*GX(KK)*
1DMY(KK)+012(K)
P22(K)=P22(KK)*(1.0-GY(KK)*DMY(KK))-P12(KK)*GY(KK)*
1DMX(KK)+022(K)
           DG=P11(K)空DMX(K)空途2+2。0空P12(K)空DMX(K)室DMY(K)+1P22(K)空DMY(K)空空2+RXTEND
GX(K)=(P11(K)室DMX(K)+P12(K)空DMY(K))/DG
GY(K)=(P12(K)空DMX(K)+P22(K)空DMY(K))/DG
AT1=(XT(KK)-SLO(K))空COS(YT(KK))
              AT2=YT(KK)-SLA(K)
TX(K)=ATAN2(AT1.AT2)
ER(K)=THETA(K)-TX(K)
1 00000
         CORRELATION GATING SCHEME TO ESTIMATE WHETHER FILTERED BEARING ANGLE THTD(K) IS AN EMISSION FROM EMITTER TARGET NTAR = I OR A SPURIOUS EMISSION
              GATE(K) = SORT(P11(K) + RXTEND) * TSTCUT
              IF(ABS(ER(K)).LT.GATE(K)) GO TO 11
IF(IFLAG.EQ.1) GO TO 3
   2
              IF(IFLAG.EQ.2) GO
GO TO 7
                                                   TO
              IF(THETAD(KK).LT.10.0.AND.THETAD(K).GT.350.0) GO IF(THETAD(KK).GT.350.0.AND.THETAD(K).LT.10.0) GO
   3
                    TO 9
              GO
              THETA(K)=THETA(K)-6.283186
    4
              IFLAG=2
```



```
GO TO 1
          THETA(K)=THETA(K)+6.283186
  5
           IFLAG=3
          GO TO 1
          THETA(K)=THETA(K)+6.283186
 6
               TO
          60
          THETA(K)=THETA(K)-6.283186
          IFLAG=1
 8
          NJ=NJ+1
           JSET (NTAR+1.NJ)=K
          NST(NTAR+1)=NJ
0000
     THTD(K) IS A SPURIOUS BEARING LINE AND IS ASSIGNED TO A
     NEW TARGET.
          NST(I) = NST(I) - 1
          NSTA=NST(I)
IF(NSTA-LT-JJ)
                                   GO TO 13
          DO 10 L=JJ.NSTA2
          JSET(I,L)=JSET(I,L+1)
GO TO 20
 10
 11
          IFLAG=1
     COMPUTE GAINS FOR LINEAR FILTERS
        EP11(K)=EP11(KK)*(1.0-G1(KK))+2.0空EP12(KK)*T(K)-1(EP12(KK)*G1(KK)+EP11(KK)*G2(KK))*T(K)+(EP22(KK)-2EP12(KK)*G2(KK))*T(K)****2+Q11(K)
        EP12(K)=EP12(KK)*(1.0-G1(KK))+(EP22(KK)-EP12(KK)*
2G2(KK))*T(K)+Q12(K)
EP22(K)=EP22(KK)-EP12(KK)*G2(KK)+Q22(K)
G1(K)=EP11(K)/(EP11(K)+RXTEND)
          G2(K) = EP12(K)/(EP11(K) + RXTEND)
0000
     FIND BEST ESTIMATES OF TARGET LONGITUDE, XTD1, AND TARGET LATITUDE, YTD1
          YTDHAT(K)=YTD(KK)+YTDDOT(KK)*T(K)
EY(K)=YTD1(KK)-YTDHAT(K)
YTD(K)=YTDHAT(K)+G1(K)*EY(K)
          YTDDOT(K)=YTDDOT(KK)+G2(K)#EY(K)
          YTD1(K)=YTD(K)+GY(K)≈ER(K)
          YT(K)=YTD1(K)/PIRAD
          XTDHAT(K)=XTD(KK)+XTDDOT(KK)*T(K)
EX(K)=(XTD1(KK)-XTDHAT(K))*COS(YT(K))
          XTD(K)=XTDHAT(K)+G1(K)*EX(K)
XTDOOT(K)=(XTDOOT(KK)+G2(K)*EX(K))*COS(YT(K))
XTD1(K)=XTD(K)+GX(K)*ER(K)
XT(K)=XTD1(K)/PIRAD
CONTINUE
 12
          WRITE(6.51)
         DO 14 JJ=J.NSTA

K=JSET(I,JJ)

WRITE(6.52)K.P11(K).P12(K).P22(K).GX(K).GY(K)

WRITE(6.50)

DO 15 JJ=J.NSTA

K=JSET(I,JJ)

WRITE(6.53)K.DMY(K).DMY(K).TY(K).ER(K).YTD(K)
 14
          WRITE(6.53)K.DMX(K).DMY(K),TX(K),ER(K),XTD(K),YTD(K)WRITE(6.56)WRITE(6.54)
 15
          DO 16 JJ=J.NSTA
K=JSET(I.JJ)
WRITE(6.52)K.EP11(K).EP12(K).EP22(K).G1(K).G2(K)
WRITE(6.55)
 16
          DO 17 JJ=J.NSTA
K=JSET(I.JJ)
WRITE(6.53)K.EX(K).EY(K).XTDDOT(K).YTDDOT(K).XTD1(K).
 17
         1YTD1(K)
                      //. 22X. 'K'. 4X. 'DMX', 6X. 'DMY', 6X, 'TX', 8X. 'XTD'. 7X, 'YTD'./)
//. 22X. 'K'. 5X. 'P11', 8X, 'P12', 8X, 'P22',
                                                      "DMX"
                                                                  6X, 'DMY', 6X, 'TX',
 50
 51
          FORMAT (
```



```
17X, 'GX', 6X, 'GY', /)

FORMAT (20X, I3, 3E11.3, 3F8.3)

FORMAT (20X, I3, 4F9.4, F11.4, F10.4)

FORMAT (//, 22X, 'K', 4X, 'EP11', 7X, 'EP12', 7X, 'EP22', 17X, 'G1', 6X, 'G2', /)

FORMAT (//, 22X, 'K', 5X, 'EX', 7X, 'EY', 4X, 'XTDDOT', 3X, 1'YTDDOT', 6X, 'XTD1', 6X, 'YTD1', /)

FORMAT ('1', /////, 34X, 'EXTENDED KALMAN FILTER', 1'PARAMETERS', //)

FORMAT ('1', /////, 25X, 'THE EXTENDED KALMAN FILTER', 1'IS INITIATED WITH THE ',/, 25X, 'FOLLOWING ELLIPSE', 2'FOR CUT NUMBER', I3, 'OF TARGET', I3)

FORMAT (//, 25X, 'THE ANGLE BETWEEN THE MAJOR AXIS AND', 2'THE MERIDIAN', 25X, 'THROUGH THE CENTER OF THE ', 3'ELLIPSE IS ', F7.3, 'DEGREES.', //, 25X, 'THE LENGTH OH', 25X, 'THE LENGTH OF THE ', 3'ELLIPSE IS ', F7.3, 'DEGREES.', //, 25X, 'THE LENGTH OF THE SEMI-MAJOR AXIS IS ', F7.3, 'DEGREES.', //, 25X, 5'THE LENGTH OF THE SEMI-MINOR AXIS IS ', F7.3, 'DEGRE', 6'ES.', //)

RETURN END
```



```
00000000
            SUBROUTINE PREPARE
            SUBROUTINE TO ESTABLISH THE VECTOR CROSS PRODUCT MATRIX FOR SUBROUTINE POSIT
            SUBROUTINE PREPAR (KI,K,SLAD,SLOD,THTD1,THTD,TLAD,
          1TLOD)
  C
            DIMENSION SLAD(1), SLOD(1), ACLAT(2), ACLON(2), THD(2)
  C
            DATA PIRAD/57#29578/
            THT1=THTD1
            THT=THTD
  CCC
       DETERMINE DIRECTION OF TRACK
            DELLAD=SLAD(K) -SLAD(KI)
DELLON=SLOD(K) -SLOD(KI)
            IF(DELLAD.EQ.O.O.AND.DELLON.EQ.O.O) GO TO 37
TRACK=90.-ATAN2(DELLAD.DELLON)* PIRAD
IF(TRACK.LT.O.O) TRACK=TRACK+360.
IF(THT.LT.TRACK.AND.THT1.GE.TRACK) GO TO 37
IF(THT1.LT.TRACK.AND.THT.GE.TRACK) GO TO 37
 CCC
       DETERMINE IF TARGET IS LEFT OR RIGHT OF TRACK
            M=1
           N=2
IF(TRACK.LE.180) GO TO 38
IF(THT1.LT.TRACK.AND.THT1.GE.(TRACK-180.)) GO TO 40
GO TO 39
CT TRACK.AND.THT1.LE.(TRACK+180.)) GO TO 39
   38
            GO TO 40
  CCC
       TARGET IS TO RIGHT OF TRACK
            IF(THT1.GT.270.AND.THT.LT.90.)
IF(THT.GT.270.AND.THT1.LT.90.)
IF(THT1.GT.THT) GO TO 37
   39
                                                                 THT = THT + 360
                                                                 THT1=THT1+360.
            M=2
            N=1
            GO TO 41
 000
       TARGET IS TO LEFT OF TRACK
            IF(THT1.GT.270.AND.THT.LT.90.)
IF(THT.GT.270.AND.THT1.LT.90.)
IF(THT.GT.THT1) GO TO 37
-ACLAT(M-)=SLAD(KI)
ACLAT(N)=SLAD(KI)
   40
                                                                 THT=THT+360.
                                                                 THT1=THT1+360.
   41
            ACLON(M) = SLOD(KI)
            ACLON(N)=SLOD(K)
THD(M)=THT1
THD(N)=THT
            CALL POSIT (ACLAT, ACLON, THD, TLAD, TLOD)
            RETURN
C
C
C
37
       BEARINGS DO NOT CROSS SO NO SOLUTION IS COMPUTED
            TLAD=0.0
            TLOD=0.0
            RETURN
  C
            END
```



```
SUBROUTINE POSIT (SLA, SLO, THETA, TLA, TLO)
         SUBROUTINE TO COMPUTE TARGET POSITION COORDINATES USING VECTOR METHODS
         DESCRIPTION OF CALLING ARGUMENTS
                     AIRCRAFT LATITUDE
AIRCRAFT LONGITUDE
BEARING ANGLES-OF-ARRIVAL
TARGET LATITUDE
TARGET LONGITUDE
         SLA
         SLA
         THETA
         TLĀ
         TLO
         SUBROUTINE POSIT (SLA, SLO, THETA, TLA, TLO)
C
         DIMENSION A(2), B(2), C(2), SLA(2), SLO(2), THETA(2)
C
         DATA PIRAD/57.29578/
DO 1 I=1.2
PH=SLA(I)/PIRAD
TH=SLO(I)/PIRAD
         OB=THETA(I)/PIRAD
ST=SIN(TH)
         CT=COS(TH)
SP=SIN(PH)
         CP=COS(PH)
CO=COS(OB)
SO=SIN(OB)
000
    COMPUTE NORMAL TO BEARING PLANE
         DX=-SP*CO*CT-ST*SO
DY=SO*CT-SP*CO*ST
         DZ = CP CO
    COMPUTE BEARING VECTORS
         X1=CP*CT
         Y1=CP&ST
         AA=Y1 DZ-SP DY
         BB=SP#DX-X1°DZ
CC=X1*DY-Y1°DX
         D=SQRT(AA率率2+BB率率2+CC率率2)
         A(I) = AA/D
         B(I) = BB/D
         C(I) = CC/D
         CONT INUE
 1
    COMPUTE TARGET POSITION VECTOR IN (X,Y,Z) COORDINATES
         X1=B(1)年C(2)-C(1)年B(2)
X2=C(1)年A(2)-A(1)年C(2)
X3=A(1)年B(2)-B(1)年A(2)
D=SQRT(X1年在2+X2年度2+X3年度2)
         X1=X1/D
X2=X2/D
         X3 = X3/D
CCC
    COMPUTE TARGET POSITION VECTOR IN (0,0) COORDINATES
         TLO=ATAN2(X2.X1)中PIRAD
TLA=ATAN2(X3.SQRT(X1中24X2中21)中PIRAD
C
         RETURN
         END
```



```
WE POINTS
NE TO LOCATE THE INTERSECTION OF THE CONES OF SOCIATED WITH EACH OF BEARING AND DESCRIBED BY RIANCE MATRICES OF THE ANGLE FILTER AND THE GFILTER
WE POINTS(I.J.SIGMA, RCUT, D11, HDGD, JSET, P11, D, THTD1, THTD.PTLAT, PTLON, JDIM, *)
N ACLAT(2).ACLON(2).D11(1).HDGD(1).JSET(JDIM.
1.PTLAT(4.JDIM.1).PTLON(4.JDIM.1).SLAD(1).
[HD(2).THTD1(1).THTD(1)
AD/57.29578/
(ABS(D11(K)) # SIGMA

P11(K) # SIGMA

102)
[01) K, THT, P11 (K), P1, THT1, D11 (K), D1
SURE THAT THE TWO BEARING LINES DO NOT ITHE SAME POINT
LAD(K) -SLAD(KI)
LAD(K) -SLOD(KI)
LOD(K) -SLOD(KI)
D.EO.O.O.AND.DELLON.EQ.O.O) GO TO 6
SUBSCRIPTS SUCH THAT PTLAT(1), PTLON(1 HE FLIGHT PATH THAN PTLAT(2), PTLON(2).
                                                           PTLON(1) IS
LE.HDGD(K)) GO TO 1
=SLAD(KI)
=SLOD(KI)
=SLAD(K)
=SLOD(K)
LE.1.) GO TO 4
EDGES OF THE ERROR ELLIPSES DO NOT CROSS, GREATER THAN 1, REDUCE IT TO 1 AND COMPUTE SECTION POINT.
104)
OF INTERSECTION OF THTD1 -- D1 WITH THTD + P1
HT1-D1
HT+P1
1.LE.THD(2).AND.M.EQ.1) GO TO
1.LE.THD(1).AND.M.EQ.2) GO TO
IT(ACLAT, ACLON, THD, TLAT, TLON)
I.J)=TLAT
 1,J)=TLON
 OF INTERSECTION OF THTD1 + D1 WITH THTD - P1
```



```
C
                THD(1)=THT1+D1
THD(2)=THT-P1
IF(THD(1).LE.THD(2).AND.M.EQ.1) GO TO
IF(THD(2).LE.THD(1).AND.M.EQ.2) GO TO
CALL POSIT(ACLAT.ACLON.THD.TLAT.TLON)
PTLAT(N.I.J)=TLAT
PTLON(N.I.J)=TLON
         FIND POINT OF INTERSECTION OF THTD1 - D1 WITH THTD - P1
                 THD(1)=THT1-D1
                 THD(2) = THT - P1
                CALL POSIT (ACLAT, ACLON, THD, TLAT, TLON)
PTLAT(3.I.J)=TLAT
PTLON(3.I.J)=TLON
        FIND POINT OF INTERSECTION OF THTD1 + D1 WITH THTD + P1
                 THD(1) = THT1 + D1
                THD(2)=THT+P1
CALL POSIT(ACLAT.ACLON,THD,TLAT,TLON)
PTLAT(4.I.J)=TLAT
PTLON(4.I.J)=TLON
                 RETURN
                WRITE(6.105)
                RETURN 1
                WRITE(6.103) P11(K),I,J
RETURN 1
  5
                WRITE(6.106) I
RETURN 1
  6
C
             FORMAT(I5,6F12.4)
FORMAT('1', /, 4X. 'K'. 5X. 'THTD(K)'. 6X. 'P11(K)'.

18X. 'P1'. 6X. 'THTD1(K)'.6X. 'D11(K)'. 8X. 'D1'. /)
FORMAT(//, 5X. 'THE ERROR COVARIANCE IS '. F12.4. '.',

1' A SOLUTION WOULD BE MEANINGLESS. I=', I3, 'J=', I3, /
FORMAT(/, 'SIGMA=1.'./)
FORMAT(//, 5X. 'THE TWO OUTSIDE BEARINGS DO NOT',

1'CROSS. SO NO SOLUTION IS POSSIBLE.', //)
FORMAT(//. 5X. 'THE BEGINNING AND END OF THE TRACK',

1'CORRELATED WITH TRAGET ', I2, 'COINCIDE, SO NO SOL',

2'UTION IS POSSIBLE', //)
RETURN
   101
   102
   103
  104
105
   106
                 RETURN
                 END
```



```
c
                    SUBROUTINE PIC
                    SUBROUTINE TO PLOT THE DF CUTS, ERROR ELLIPSES COMPUTED FROM THE ANGLE FILTER AND THE ERROR ELLIPSES DESCRIBED BY THE COVARIANCE MATRIX OF THE EXTENDED KALMAN FILTER
                    MODES
                                            PLOTS THE
PLOTS THE
TERMS FROM
TED WITH CUT
                                                                                   DF CUTS CORRELATED TO TARGET I ERROR ELLIPSE DESCRIBED BY THE THE ANGLE FILTERING ROUTINE J OF TARGET I
                    IFLAG=1
IFLAG=2
                     AND D11
                     ASSOCIATED
                    THIS SUBROUTINE FIRST DETERMINES THE MAJOR DIRECTION OF THE PLOT, PLOTS IT THE MAJOR DIRECTION OF THE PLOUSING A MERCATOR PROJECTION
                                                                                                                                                                                        PLOT
                 SUBROUTINE PIC(IFLAG.I, JJ, NST, JSET, SLAD, SLOD, TLAD, 1TLOD, THETA, THTD, THTD1, P11, D11, HDGD, RCUT, JDIM, PTLAT, 2PTLON, SIGMA, P12, P22, XTD1, YTD1)
C
                DIMENSION ACLAT(100),ACLON(100),A(4),B(4),D11(1),
1FLAD(100),FLOD(100),HDGD(1),IT1(12),IT2(12),IT3(12),
2IT4(13),JSET(JDIM,1),NST(1),NUMB(25),PTLAT(4,JDIM,1),
3PTLON(4,JDIM,1),P11(1),SLAD(1),SLOD(1),THETA(1),
4TLAD(1),TLOD(1),THTD(1),THTD1(1),XX(101),XZ(101),
5YY(101),P12(1),P22(1),XTD1(1),YTD1(1),IT5(13)
                                                                                          CUTS AND
EDWARD H.
NTAR
C
                                    IT1/'PLOT OF DF IT2/'
IT3/'
                                                                                                             AND A/C NAVIGATION DATA D_H. MILLS
                    DATA
                    DATA
                DATA IT4/'PLOT', 'OF ', 'ERRO', 'R CO', 'VARI', 'ANCE',

1'S OF', 'SMO', 'OTHE', 'D BE', 'ARIN', 'G LI', 'NES '/

DATA IT5/'PLOT', 'OF ', 'ERRO', 'R CO', 'VARI', 'ANCE',

1'S OF', 'EXT', 'ENDE', 'D KA', 'LMAN', 'FIL', 'TER '/

DATA NUMB/'1', '2', '3', '4', '5', '6', '7', '8', '9', '10',

1'11', '12', '13', '14', '15', '16', '17', '18', '19', '20',

2'21', '22', '23', '24', '25'/
C
                     DATA PIRAD/57.29578/
C
                    NSTA=NST(I)
IF(IFLAG.EQ.1) J
DO 1 J=1.NSTA
K=JSET(I.J)
ACLAT(J)=SLAD(K)
ACLON(J)=SLOD(K)
IF(IFLAG.EQ.2) G
IF(IFLAG.EQ.3) G
A(1)=TLAD(I)
B(1)=TLAD(I)
                                                                        JJ=NSTA
    1
                                                                       GO
                                                                       GO
                    B(1)=TLOD(1)
ALATS=SMALL2(ACLAT, NSTA, A, 1)
ALATH=HUGE2(ACLAT, NSTA, A, 1)
ALONS=SMALL2(ACLON, NSTA, B, 1)
ALONH=HUGE2(ACLON, NSTA, B, 1)
                 GO TO 12
IF(NSTA.LE.3) GO TO 1000
CALL POINTS(I.JJ.SIGMA.RCUT,D11.HDGD.JSET,P11,SLAD,
1SLOD.THTD1.THTD.PTLAT,PTLON,JDIM,&1000)
DO 10 N=1.4
XX(N)=PTLON(N.I.JJ)
YY(N)=PTLAT(N,I.JJ)
CO 10 42
    10
                     GO TO 42
K=JSET(I,JJ)
CALL ELIPS7(P11(K),P12(K),P22(K),XTD1(K),YTD1(K),XX,
    41
                  CALL
1YY)
```



```
N = 101
           ALONS=SMALL2(ACLON, NSTA, XX, N)
ALONH=HUGE2(ACLON, NSTA, XX, N)
ALATS=SMALL2(ACLAT, NSTA, YY, N)
ALATH=HUGE2(ACLAT, NSTA, YY, N)
  42
0000
       ADAD=AIRCRAFT POSITION LATITUDE VARIATION IN DEGREES ADOD=AIRCRAFT POSITION LONGITUDE VARIATION IN DEGREES
           ACAD=ABS(ALATH-ALATS)
ADOD=ABS(ALONH-ALONS)
ASLAR=((ALATH+ALATS)/2.)/PIRAD
CSLA=COS(ASLAR)
  12
            ADAD2=.75%ADAD
            IF(ADOD.GT.ADAD2) GO TO 21
000000000
        ADAD IS GREATER THAN ADOD
       PROGRAM TO SCALE DATA FOR NORTH LATITUDE AND WEST LONGITUDE ONLY
        COMPUTE X AND Y
                                         SCALE INCREMENT VALUES DX AND
           DX=1.

1F(ADAD.GT.8) DX=ADAD/8.

IF(ADOD.GT.6) DX=ADOD/6.

IF(ADOD.LE.3.AND.ADAD.LE.4) DX=.5

IF(ADOD.LE.1.5.AND.ADAD.LE.2) DY=.25
00000
        COMPUTE X AND Y AXIS
                                                 INITIAL VALUES XMIN AND YMIN
        LONGITUDE IS WEST (-)
                                                IN DEGREES
           ABSC=0.0
MIN=ALONS-(DX+.5)
            XMIN=MIN.
000
        LATITUDE IS NORTH(+) IN DEGREES
           ORD=90.0
           MIN=ALATS-(DY+.5)
            VMIN=MIN
000
         DRAW AXIS
            XDRIG=0.0
           YORIG=0.0
           CDIV=1
            SDIV=CSLA
            LA=1
           LB=2
           LC=-7
IT3(7)=NUMB(I)
         TI3(7)=NUMB(1)
CALL PLOTS
CALL PLOT(-3.,0.,-3)
CALL PLOT(1.,0.,-3)
CALL PLOT(1.,0.,-3)
CALL AXIS(XORIG.YORIG.'DEGREES LONGITUDE',-17,6.0,

1ABSC.XMIN.DX.CDIV.SDIV)
CALL AXIS(XORIG.YORIG.'DEGREES LATITUDE',-16,8.,ORD,

1YMIN.DY.CDIV.SDIV)
CALL SYMBOL(0.0.14.0.0.21.IT2,0.0.40)
CALL SYMBOL(0.0.13.5.0.21.IT3,0.0.40)
CCC
        SCALE DATA
            DY=DX CSLA
            DO 2 N=1.NSTA
            XX(N) = (ACLON(N) - XMIN)/DX
            YY(N) = (ACLAT(N) - YMIN)/DY
          DRAW NAV TRACK
```



```
C
             CALL LINE(XX, YY, NSTA, LA, LB)
  C
            IF(IFLAG.EQ.2) GO TO 5
IF(IFLAG.EQ.3) GO TO 43
CALL SYMBOL(0.0,14.5,0.21,IT1,0.0,40)
  CCC
         PLOT DF CUTS TO FORM THEORETICAL EMMITTER LOCATION
             DO 4 J=1.NSTA
            DT=0.0
K=JSET(I.J)
CTH=COS(THETA(K))
STH=SIN(THETA(K))
DO 3 L=1.100
             LL=L
             FLAD(L)=YY(J)+DT*CTH
FLOD(L)=XX(J)+DT*STH
IF(FLAD(L).GT.8 .OR.FLAD(L).LT.0),GO TO
             IF(FLOD(L).GT.6. OR. FLOD(L).LT.0) GO TO 13
            DT=.07% L
CONTINUE
CALL LINE(FLOD, FLAD, LL, LA, LC)
313
4
CCCCCCC
             CONTINUE
         SUBROUTINE TO PLOT SMOOTHED INITIAL BEARING LINE AND KALMAN FILTERED FINAL BEARING LINE
         SMOOTHED INITIAL BEARING LINE (THTD1)
            KI=JSET(I.1)
KF=JSET(I.NSTA)
THT1=THTD1(KF)/PIRAD
            FX=XX(1)+8.* SIN(THT1)
FY=YY(1)+8.* COS(THT1)
SX=XX(1)
SY=YY(1)
            CALL PLOT(SX.SY.3)
CALL PLOT(FX.FY.2)
  CCC
         KALMAN FILTERED FINAL BEARING LINE (THTD)
            THTF=THTD(KF)/PIRAD
            FY=YY(NSTA)+8. COS(THTF)
FX=XX(NSTA)+8. SIN(THTF)
SX=XX(NSTA)
SY=YY(NSTA)
            CALL PLOT(SX.SY.3)
CALL PLOT(FX.FY.2)
  C
            GO TO 8
CCCC 5
         PLOT THE ELLIPSE FORMED BY THE P11 TERMS OF THE ERROR COVARIANCE MATRICES.
                    SYMBOL(0.0,15.0,0.21,IT4,0.0,52)
             FLOOT=JJ
            CALL NUMBER(5.25, 7...14, FLOOT, 90.,-1)
KI=JSET(I.1)
K=JSET(I.JJ)
           DO 11 N=1.4

XX(N)=PTLON(N,I,JJ)

YY(N)=PTLAT(N,I,JJ)

CALL ELIPS6(KI.K,SLAD,SLOD,THTD1(K),THTD(K),D11(K),

1P11(K).XX,YY.A1.B1,ALPHA1,TLAD(I),TLOD(I))
    11
             DO 6 K=1.101
    20
             XX(K) = (XX(K) - XMIN) / DX

YY(K) = (YY(K) - YMIN) / DY
    6
             CALL LINE(XX,YY,101,1,1)
  CCC
         PLOT THE FILTERED FIRST AND CURRENT FINAL CUTS.
```



```
K=JSET(I.JJ)
THT1=THTD1(K)/PIRAD
SX=(ACLAT(1)-XMIN)/DX
SY=(ACLON(1)-YMIN)/DY
FX=SX+8.*COS(THT1)
FY=SY+8.*SIN(THT1)
CALL PLOT(SX.SY.3)
CALL PLOT(FX.FY.2)
THT=THTD(K)/PIRAD
SX=(ACLAT(J)-XMIN)/DX
SY=(ACLCN(J)-YMIN)/DY
FX=SX+8.*COS(THT)
FY=SY+8.*SIN(THT)
CALL PLOT(FX.FY.2)
CALL PLOT(FX.FY.2)
C
              GO TO 8
0000
         PLOT THE ELLIPSE FORMED BY THE ERROR COVARIANCE TERMS OF THE EXTENDED KALMAN FILTER.
  43
                        SYMBOL (0.0, 15.0, 0.21, IT5, 0.0, 52)
              FLOOT=JJ
              CALL
                        NUMBER (5.25, 7.,.14, FLOOT, 90.,-1)
              K=JSET (I.JJ)
            CALL ELIPS7(P11(K), P12(K), P22(K), XTD1(K), YTD1(K), XX, 1YY)
              DO 44 K=1.101

XX(K)=(XX(K)-XMIN)/DX

YY(K)=(YY(K)-YMIN)/DY

CALL LINE(XX.YY.101.1.1)
  44
C
              CALL PLOT(-3..18..-3)
CALL PLOT(1.75.0..-3)
  8
                        PLOTE
              CALL
              RETURN
C
  21
              DY=1.
000
       ADOD IS GREATER THAN ADAD
         IF(ADOD.GE.8) DY=ADOD/8.
IF(ADAD.GT.6) DY=ADAD/6.
IF(ADOD.LE.4.AND.ADAD.LE.3.) DY=.5
IF(ADOD.LE.2.AND.ADAD.LE.1.5) DY=.25
LONGITUDE IS WEST(-) IN DEGREES
C
              DX=DY
              ABSC=90.0
MIN=ALONS-(DY+.5)
              YMIN=MIN
         IF(DY.LT.(.5)) YMIN=ALONS-DY
LATITUDE IS NORTH(+) IN DEGREES
C
              ORD=0.0
             MIN=ALATS-.5

XMIN=MIN

IF(ABS(XMIN-ALATS).GT..9)

IF(DX.LT.(.5)) XMIN=ALATS

XMAX=XMIN+5.*DX
                                                                         XMIN=XMIN+1.
              DX = -DX
           DRAW AXIS
              XORIG=0.0
YORIG=0.0
CDIV=CSLA
              SDIV=1.
              LA = 1
              LB=2
              LC=-7
IT3(7)=NUMB(I)
CALL PLOTS
              CALL PLOT (-3., 0., -3)
```



```
CALL PLOT(2..0..-3)
CALL AXIS(XORIG.YORIG.'DEGREES LATITUDE'.-16.6.0.ORD.
1XMAX.DX.CDIV.SDIV)
CALL AXIS(XORIG.YORIG.'DEGREES LONGITUDE'.-17.8.0.
1ABSC.YMIN.DY.CDIV.SDIV)
CALL PLOT(XORIG.YORIG.3)
CALL PLOT(0.0.YORIG.2)
CALL PLOT(0.0.0.0.2.2)
            CALL PLOT(0.0.0.0.2)
CALL SYMBOL(0.0.14.0.0.21.IT2.0.0.40)
CALL SYMBOL(0.0.13.5.0.21.IT3.0.0.40)
            CALL PLOT (XORIG, 0., -3)
CCC
        SCALE DATA
            DX=DX&CSLA
DO 22 N=1.NSTA
XX(N)=(ACLAT(N)-XMIN)/DX
YY(N)=(ACLON(N)-YMIN)/DY
  22
000
          DRAW NAV TRACK
            CALL LINE(XX,YY,NSTA,LA,LB)
C
            IF(IFLAG.EQ.2) GO TO 27
IF(IFLAG.EQ.3) GO TO 45
ORIGX=XORIG-.75
ORIGY=YURIG+.3
CALL_SYMBOL(-ORIGX,ORIGY..14,IT1,0.,40)
            FLOOT=I
            CALL NUMBER (-. 75, 7., . 14, FLOOT, 90., -1)
CCC
        PLOT OF CUTS TO FORM THEORETICAL EMMITTER LOCATION
            DO 26 J=1.NSTA
            DT=0.0

K=JSET(I.J)

CTH=COS(THETA(K))

STH=SIN(THETA(K))

DO 23 L=1.100
            LL=L
            FLAD(L)=XX(J)-DT%CTH
FLOD(L)=YY(J)+DT%STH
IF(FLAD(L).GT.O.OR.FLAD(L).LT.-6) GO TO 24
IF(FLOD(L).GT. 8. OR. FLOD(L).LT.0) GO TO
            DT = .07 L
CONTINUE
  23
24
26
            CALL LINE(FLAD, FLOD, LL, LA, LC)
            CONTINUE
        SUBROUTINE TO PLOT SMOOTHED INITIAL BEARING LINE AND
        KALMAN FILTERED FINAL BEARING LINE
        SMOCTHED INITIAL BEARING LINE (THTD1)
            KF=JSET(I.NSTA)
THT1=THTD1(KF)/PIRAD
FX=XX(1)+8.*CDS(THT1)
FY=YY(1)+8.*SIN(THT1)
            SX=XX(1)
SY=YY(1)
            CALL PLOT(SX.SY.3)
CALL PLOT(FX.FY.2)
            WRITE(6,101) SX,SY
CCC
        KALMAN FILTERED FINAL BEARING LINE (THTD)
            THTF=THTD(KF)/PIRAD

FX=XX(NSTA)-8. COS(THTF)

FY=YY(NSTA)+8. SIN(THTF)

SX=XX(NSTA)

SY=YY(NSTA)
            CALL PLOT(SX,SY,3)
```



```
CALL PLOT(FX.FY.2)
WRITE(6.101) SX.SY
GO TO 30
C
C
C
C
27
           PLOT THE ELLIPSE FORMED BY THE P11 TERMS OF THE
           ERROR COVARIANCE MATRICES.
                ORIGY=YORIG+.3
                CALL SYMBOL(-XORIG, ORIGY, 0.14, IT4, 0., 52)
                FLOOT=JJ
               CALL NUMBER(-.75,7.,0.14,FLOOT,90.,-1)
KI=JSET(I,1)
K=JSET(I,J)
             DO 40 N=1,4

XX(N)=PTLON(N,I,JJ)

YY(N)=PTLAT(N,I,JJ)

CALL ELIPS6(KI,K,SLAD,SLOD,THTD1(K),THTD(K),D11(K),

1P11(K),XX,YY,A1,B1,ALPHA1,TLAD(I),TLOD(I))

WRITE(6,101)(XX(J),YY(J),J=1,10)

WRITE(6,101) XMIN,YMIN

DO 29 J=1,101

XZ(J)=(YY(J)-XMIN)/DX

YY(J)=(XX(J)-YMIN)/DY

WRITE(6,101)(XZ(J),YY(J),J=1,10)

CALL LINE(XZ,YY,101,1,1)
                DO 40 N=1,4
     40
     29
           PLOT THE FILTERED FIRST AND CURRENT FINAL CUTS.
                THT1=THTD1(K)/PIRAD
SX=(ACLAT(1)-XMIN)/DX
SY=(ACLON(1)-YMIN)/DY
                FX=SX-7. COS(THT1)
               FX=SX-7.%COS(THT1)
FY=SY+7.%SIN(THT1)
CALL PLOT(SX.SY.3)
CALL PLOT(FX.FY.2)
THT=THTD(K)/PIRAD
WRITE(6.100) K.THT
SX=(ACLAT(JJ)-XMIN)/DX
SY=(ACLAT(JJ)-YMIN)/DY
FX=SX-7.%COS(THT)
FY=SY+7.%SIN(THT)
CALL PLOT(SX.SY.3)
               CALL PLOT(SX,SY,3)
CALL PLOT(FX,FY,2)
GO TO 30
  2000
           PLOT THE ELLIPSE FORMED BY THE ERROR COVARIANCE TERMS
                 THE EXTENDED KALMAN FILTER.
    45
                ORIGY=YORIG+.3
                CALL SYMBOL (-XORIG, ORIGY, 0.14, IT5, 0., 52)
                FLOOT=JJ
               CALL NUMBER(-.75, 7...14.FLOOT,90.,-1)
K=JSET(I.JJ)
CALL ELIPS7(P11(K),P12(K),P22(K),XTD1(K),YTD1(K),XX,
              1YY)
               DO 46 J=1.101
XZ(J)=(YY(J)-XMIN)/DX
YY(J)=(XX(J)-YMIN)/DY
     46
                CALL LINE(XZ, YY, 101, 1, 1)
MARK THE KNOWN POSITION OF THE TARGET
               IF(I.GT.2) GO TO 47
IF(I.EQ.2) GO TO 36
ATLAD=33.24055
ATLON=-117.4169
GO TO 37
ATLAD=32.78222
ATLON=-117.2244
SX1=(ATLAD-XMIN)/DX
     36
    37
                SY1 = (ATLON-YMIN)/DY
                SX = SX1 - 1.
```



```
FX=SX1+1.

SY=SY1-1.

FY=SY1+1.

CALL PLOT(SX.SY1.3)

CALL PLOT(FX.SY1.2)

CALL PLOT(SX1.SY.3)

CALL PLOT(SX1.FY.2)

CALL PLOT(-12..18..-3)

CALL PLOT(1.75.0..-3)

CALL PLOTE
  47
               CALL
                            PLOTE
               FORMAT(15,2F12.6)
FORMAT(2F12.6,/)
FORMAT('1', I3)
FORMAT(2F12.3,3I5)
  1.00
  101
102
  105
               RETURN
  1000
               END
00000000000
               SUBROUTINE AXIS
               SUBROUTINE TO COMPUTE. DRAW AND LABEL THE AXES FOR A MERCATOR PROJECTION IN THE NORTH WEST HEMISPHERE
            SUBROUTINE AXIS (XB,YB,BCD,NC,SIZE,THETA,YMIN,DY,CDIV, 1SDIV)
               DIMENSION BCD(1)
               ZING=1.0
IF(NC)1.2.2
               ZING = -1.0
NAC=IABS(NC)
TH=THETA:.0174533
CTH=COS(TH)/CDIV
               STH=SIN(TH)/SDIV
N = SIZE+0.50
IF(THETA.EQ.90.AND.CDIV.EQ.1) N=SIZE*SDIV
IF(THETA.EQ.0.AND.SDIV.EQ.1) N=SIZE*CDIV
               TN=N
               X=XB
Y=YB
               XA = X - 0.1 * ZING *
YA = Y + 0.1 * ZING *
CALL PLOT (XA, YA, 3)
               CALL PLOT (XA, YA, 3)
DO 20 I=1, N
CALL PLOT (XB, YB, 2)
XC=XB + CTH
YC=YB+STH
               CALL PLOT
                                        (XC, YC, 2)
               XA=XA+CTH
YA=YA + STH
CALL PLOT(XA,YA,2)
XB=XC
               YB=YC

YA=Y-0.250

IF(ABS(YMIN).GE.100.)YA=Y-.375

IF(ABS(YMIN).GE.100.AND.YMIN.LT.0.0) YA=Y-.50

IF(THETA.E0.90.0) GD TD 22
  20
              IF(IHETA.EQ.90.0) GO TO 22
IF(THETA.EQ.0.0) GO TO 24

XA=X + 0.250
GO TO 25

XA=X-.250
IF(ABS(YMIN).GE.100.)XA=X-.375
IF(ABS(YMIN).GE.100.AND.YMIN.LT.0.0) XA=X-.50
WRITE(6.61)XA,YA
ABSV=YMIN
DO 30 J=1.N
  22
  24
  25
               DO 30 I=1.N
CALL NUMBER(XA.YA.O.14.ABSV,THETA.2)
               ABSV=ABSV+DY
```



```
XA=XA+CTH
YA=YA+STH
CONTINUE
TNC=NAC+7
30
                    TNC=NAC+7
CTH=CTH©CDIV
STH=STH©SDIV
XA=X+(SIZE /2.0-.06 *TNC)*CTH - (-.07 + ZING*.42)* STH
YA=Y+(SIZE /2.0-.06 *TNC)*STH + (-.07 + ZING*.42)* CTH
CALL SYMBOL(XA,YA,0.14,BCD,THETA,NAC)
WRITE(6.62)XB,YB
FORMAT('0 ABSV=',F10.5)
FORMAT('0 XA=',F10.5,' YA=',F10.5)
FORMAT(/,3X,'XB=',F10.5,2X,'YB=',F10.5,/)
RETURN
60
61
62
50
                      RETURN
                      END
                     FUNCTION SMALL2(A,NSTA,B,N)
DIMENSION A(50),B(50)
SMALL2=A(1)
DO 1 I=2.NSTA
IF(SMALL2.LE.A(I)) GO TO 1
SMALL2=A(I)
CONTINUE
DO 2 I-1.N
1
                     DO 2 I=1.N
IF(SMALL2.LE.B(I)) GO TO 2
SMALL2=B(I)
CONTINUE
2
                      RETURN
                      END
                    FUNCTION HUGE2 (A,NSTA,B,N)
DIMENSION A(50),B(50)
HUGE2=A(1)
DO 1 I=2.NSTA
IF(HUGE2.GE.A(I)) GO TO 1
HUGE2=A(I)
CONTINUE
DO 2 I=1.N
IF(HUGE2.GE.B(I)) GO TO 2
HUGE2=B(I)
CONTINUE
RETURN
1
2
                      RETURN
END
```



```
00000000
         SUBROUTINE ELIPS6
         SUBROUTINE TO DESCRIBE AN ELLIPSE TO FIT THE FOUR POINTS COMPUTED IN SUBROUTINE POINTS
         SUBROUTINE ELIPS6(KI,KC,SLAD,SLOD,THTD1,THTD,D11,P11,
       1X3.Y3.A1.B1.ALPHA1.TLAD.TLOD)
C
         DIMENSION SLAD(1), SLOD(1), X(101), X3(101), Y(101),
       1Y3(101), Z(4)
C
         DATA PIRAD/57.29578/
C
         CALL PREPAR (KI, KC, SLAD, SLOD, THTD1, THTD, TLAD, TLOD)
0000000
    TLAD. TLOD IS THE CURRENT ESTIMATE OF THE TARGET
    POSITION.
    X3.Y3 ARE THE LOCATIONS OF THE INTERSECTIONS OF THE EDGES OF THE TWO ERROR CONES.
        DELX=X3(1)-X3(2)
DELY=Y3(1)-Y3(2)
ALPHA1=ATAN2(DELX,DELY)
ALPHA2=ALPHA1-1.5737963
         ALPHAD=ALPHA1 * PIRAD
000
    ALPHAD IS THE ANGLE OF ROTATION OF THE ELLIPSE
        DELX=TLOD-X3(2)
DELY=TLAD-Y3(2)
A1=SQRT(DELX+02+DELY+2)
000
    Al
        IS THE SEMI-MAJOR AXIS
        DO 4 K=3,4
DELX=TLOD-X3(K)
DELY=TLAD-Y3(K)
         BETA = ATAN2 (DEL X. DELY)
        GAMMA=ALPHA2-BETA
Z(K)=SQRT(DELX**2+DELY**2)*COS(GAMMA)
B1=(ABS(Z(3))+ABS(Z(4)))/2.0
 4
CCC
    B1
       IS THE SEMI-MINOR AXIS
         WRITE(6.103) ALPHAD, A1, B1
C
         EPS=A1/25.
         Y(1) = -A1

Y(101) = -A1
         AA=A1 8 ≥ 2
         BB=B18 ≈ 2
        DO 1 K=2.51
KK=K-1
Y(K)=Y(KK)+EPS
         Y(102-K)=Y(K)
DO 2 K=1.51
 1
        CC=ABS(BB#(1.-(Y(K)402)/AA))
X(K)=SORT(CC)
 2
         X(102-K) = -X(K)
    ROTATE THE ELLIPSE BY ALPHAD AND TRANSLATE ITS CENTER TO
    TLOD. TLAD.
         CA=COS(ALPHA1)
        SA=SIN(ALPHA1)
DO 3 K=1,101
```



```
X3(K)=X(K)*CA+Y(K)*SA+TLOD

Y3(K)=-X(K)*SA+Y(K)*CA+TLAD

WRITE(6.101)X3(1),Y3(1).X3(50),Y3(50),X3(25),Y3(25),

1X3(75).Y3(75)

C

101 FORMAT(4F12.2)
FORMAT(//.5X.*THE ANGLE BETWEEN THE MAJOR AXIS AND ',

2'THE MERIDIAN'./.5X.*THROUGH THE CENTER OF THE ',

3'FLLIPSE IS '.F7.3.* DEGREES.*.//.5X.*THE LENGTH OF ',

4'THE SEMI-MAJOR AXIS IS '.F7.3.* DEGREES.*.//.5X,

5'THE LENGTH OF THE SEMI-MINOR AXIS IS ',F7.3.* DEGRE',

6'ES.*.//)

RETURN
END
```



```
SUBROUTINE ELIPS7
           SUBROUTINE TO DESCRIBE THE ELLIPSE TO FIT THE ERROR COVARIANCE TERMS IN THE EXTENDED KALMAN FILTER
           SUBROUTINE ELIPS7(A,C,B,TLOD,TLAD,X3,Y3)
C
           DIMENSION X(101), Y(101), X3(101), Y3(101)
C
           DATA PIRAD/57.29578/
C
           AMIB=A-B
IF(ABS(AMIB).LT.1E-8) GO TO 10
ALPHAD=90.-.5*ATAN2((2.*C),AMIB)*PIRAD
                TO 11
           GO
           ALPHAD=0.0
ALPHA1=ALPHAD/PIRAD
 10
11
           CA=COS (ALPHAI)
           SA=SIN(ALPHA1)
           A2=ABS(AASAA+2+2。中C中SA中CA+B中CA+中2)
A1=SQRT(A2)
B2=ABS(A+CA++2-2-+C中SA+CA+B中SA++2)
           B1=SQRT(B2)
0000000
     ALPHAD IS THE ANGLE OF ROTATION OF THE ELLIPSE
     A1 IS THE SEMI-MAJOR AXIS
     B1 IS THE SEMI-MINOR AXIS
           WRITE(6,103) ALPHAD, A1, B1
C
           EPS=A1/25.
           Y(1) = -A1

Y(101) = -A1
           BB=B2
           AA = A2

DO 1 K = 2.51
           KK=K-1
Y(K)=Y(KK)+EPS
           Y(102-K)=Y(K)
  1
           DO 2 K=1.51
CC=ABS(BB*(1.-(Y(K)**2)/AA))
X(K)=SQRT(CG)
           X(102-K)=-X(K)
 2
0000
       ROTATE THE ELIPSE SUCH THAT AA IS PARALLEL TO THTD1 AND TRANSLATE TO XZERO AND YZERO.
           DO 3 K=1.101
X3(K)=X(K)*CA+Y(K)*SA+TLOD
           Y3(K) =-X(K) SA+Y(K) CA+TLAD
  3
                23 K=1,51
           WRITE(6,101)X3(K),Y3(K),X3(K+50),Y3(K+50)
  23
C
         FORMAT (4F12.2)
FORMAT (5X, 'THE ANGLE BETWEEN THE MAJOR AXIS AND THE '
1'MERIDIAN THROUGH THE CENTER OF THE ELLIPSE IS ',F7.3,
1' DEGREES'./.5X, 'THE LENGTH OF THE SEMI-MAJOR AXIS'
2'IS'.F7.3.' DEGREES'./.5X, 'THE LENGTH OF THE SEMI-'
3'MINOR AXIS IS'.F7.3.' DEGREES'./)
FORMAT (/.5X,'LATTITUDE AND LONGITUDE OF', /, 3X,
1'THTD + P11 THTD -P11 THTD1 - D11 THTD1 + D11', /,3X
2'WITH THTD1 WITH THTD1 WITH THTD', /)
  101
  103
  104
           RETURN
           END
```



```
SUBROUTINE MONACO(NUM,IX,STDEV,STDEVN,AMEAN,KFLAG,NPUN
             SUBROUTINE TO READ AIRCRAFT DATA AND ADD GAUSSIANLY
             DISTRIBUTED RANDCM NOISE
             LIST OF VARIABLES
                             INITIAL NUMBER FOR RANDOM NUMBER GENERATOR STANDARD DEVIATION OF NOISE ADDED TO DE BESTANDARD DEVIATION OF NOISE ADDED TO NAVIGMEAN OF ALL NOISE.
             ΙX
             STDEV -
STDEVN-
                                                                                                           DF BEARIN
                                                                                                           NAVIGATIO
                             MEAN OF ALL NOISE
COUNTER FOR REPEATED MONTE C
NUMBER OF RUNS DESIRED FOR M
SIMULATION. MUST BE GREATER
MEAN OF DF BEARING NOISE
MEAN OF LONGITUDE NOISE
MEAN OF LATITUDE NOISE
VARIANCE OF BEARING NOISE
VARIANCE OF BEARING NOISE
             AMEAN
KELAG
                                                                                      CARLO
                                                                                                   RUNS
             NRUN
                                                                                    MONTE
                                                                                                   CARLO
                                                                                        THAN
                                                                                                         EQUAL
            AMEANT -
AMEANE-
AMEANN-
VARIT -
            VARIE
VARIN
                             VARIANCE OF LONGITUDE NOISE
VARIANCE OF LATITUDE NOISE
CONVERSION FACTOR 57.29578 DEGREES PER RADIAN
             PIRAD
             SUBROUTINE MONACO(NUM, IX, STDEV, STDEVN, AMEAN, KFLAG, NRUN
C
          COMMON ACLAD(100).ACLAMD(100).ACLAR(100).ACLOD(100).
1ACLOMD(100).ACLOR(100).ALT(100).BRNG(100).BRNGD(100).
2E(100).FREQ(100).G1(100).G2(100).GATE(100).HDG(100).
3HDGD(100).MODEN(100).MODET(100).NST(100).P11(100).
4P12(100).P22(100).PITCH(100).PRF(100).PW(100).
5ROLL(100).SLA(100).SLAD(100).SLO(100).SLOD(100).
5T(100).TIMEN(100).TIMET(100).TDTD(100).THTD(100).
6THTD1(100).THETA(100).THETAD(100).TLAD(100).TLCD(100).
7VEL(100).VELE(100).VELN(100).XTD(100).YTD(100).
C
             DATA AMEANT, AMEANE, AMEANN, VARIT, VARIE, VARIN/O.,O.,O.,
          10..0..0./
DATA PIRAD/57.29578/
C
             IF (KFLAG.NE.1) GO TO 3
00000
        SUBROUTINE TO READ EMITTER TARGET AND AIRCRAFT NAVIGATION DATA FROM CARD DATA DECK DATA SEQUENCE MUST BE OF FORMAT TGT/NAV
             DO 1 I = 1,100
             NUM = I - 1
        READ EMITTER TARGET DATA
             READ(5,48,END=8) TGT, TIMET(I), BRNGD(I), PRF(I), PW(I),
          1 FREQ(I)
000
        READ AIRCRAFT NAVIGATION DATA
             READ(5,48)HDGD(I), ACLAMD(I), ACLOMD(I), ALT(I), VELN(I),
           1 VELE(I)
CONTINUE
  1
CCC
         SUBROUTINE TO COMPUTE AIRCRAFT VELOCITY
             DO 2 I=1,NUM
VELS=VELE(I)※度2+VELN(I) 
VEL(I)=SQRT(VELS)
  8
         SUBROUTINE TO CHANGE ANGLES FROM DEGREES TO RADIANS
```



```
C
                  HDG(I)=HDGD(I)/PIRAD
                  BRNG(I)=BRNGD(I)/PIRAD
    2
                  CONT INUE
CCCC 3
            GENERATE RANDOM NOISE TO ADD TO KNOWN EMITTER BEARING ANGLES OF ARRIVAL AND AIRCRAFT POSITION FIXES.
                  IXE=5。於IX
IXN=7錄IX
                 IXN=7%1X
DO 4 I=1,NUM
CALL GAUSS(IX,STDEV,AMEAN,V)
AMEANT=AMEANT+V/NUM
VARIT=VARIT+V**2/NUM
THETAD(I)=BRNGD(I)+V
THETA(I)=THETAD(I)/PIRAD
IF(THETA(I).GT.6.283186) THETA(I)=THETA(I)+6.283186
IF(THETAD(I).GT.360.0) THETAD(I)=THETAD(I)-360.0
CALL GAUSS(IXE,STDEVN,AMEAN,V)
AMEANE=AMEANE+V/NUM
VARIF=VARIE+V** 2/NUM
                  VARIE=VARIE+V® 2/NUM
ACLOD(I)=ACLOMD(I)+V/600.
ACLOR(I)=ACLOD(I)/PIRAD
CALL GAUSS(IXN.STDEVN, AMEAN, V)
                  AMEANN=AMEANN+V/NUM
VARIN=VARIN+V® 2/NUM
ACLAD(I)=ACLAMD(I)+V/600.
                  ACLAR(I) = ACLAD(I) / PIRAD
                  CONT INUE
DEVNT=SORT (VARIT)
DEVNE=SORT (VARIE)
DEVNN=SORT (VARIE)
    4
                  IF (KFLAG. EQ. NRUN)
                                                                 GO TO 6
               IF(KFLAG.EG.NKON) GG TG G
IF(KFLAG.NE.1) GG TG 7
WRITE(6.201)
WRITE(6.49)(I,TIMET(I),FREQ(I),PRF(I),PW(I),BRNGD(I),
IV.THETAD(I).HDGD(I).ACLAMD(I),ACLOMD(I),ALT(I),
2VELN(I).VELE(I).I=1.NUM)
WRITE(6.50)
WRITE(6.51)(ACLAMD(I),ACLAD(I),ACLOMD(I),ACLOD(I),
IT-1.NUM)
               1 I = 1 . NUM)
                  WRITE(6, 202) AMEANT, DEVNT, AMEANE, DEVNE, AMEANN, DEVNN
                  KFLAG=KFLAG+1
                  RETURN
               FORMAT (6F11.5)
FORMAT (* * . I3.2X,F5.1,2X,F6.1,2X,F5.1,2X,F4.2,2X,F9.5,12X,F8.5,2X,F9.5,2X,F5.1,2X,F8.5,2X,F10.5,2X,2F8.1,
               PORMAT(3X.'LATITUDE
1'LONGITUDE'./)
     50
                                                                         NOISY LAT
                                                                                                      LONGITUDE
                                                                                                                                   NOISY .
                  FORMAT (4F11.5)
               FORMAT("1".36X, LISTING OF EMITTER TARGET DATA AND 1'AIRCRAFT NAVIGATION DATA', //, 13X, TARGET PARAMET 2.37X, AIRCRAFT PARAMETERS', //, JSET TIMET FREQ 3' PRE PW BRNGD V THETAD HOGD
    201
                                                                                                                                    PARAMETERS!
                                                                                                               THETAD HD
                         SLAD
                                                       SLOD
                                                                                                      VELN
               FORMAT( ///. 5X, "NOISE ADDED TO BEARING ANGLES OF ',
1'ARRIVAL'./.10X."MEAN = '.F5.2,/.10X, 'SIGMA = '.F5.2,
2//.5X. "NOISE ADDED TO LONGITUDE FIXES'./, 10X, 'MEAN'
3' = '.F5.2. /. 10X. 'SIGMA = ', F5.2, //. 5X, 'NOISE'
4'ADDED TO LATITUDE FIXES'./, 10X, 'MEAN = ', F5.2, /
    202
                               SIGMA =
               510X.
                                                      1. F5.2,///)
                  END
```



```
0000000
                  SUBROUTINE READ
                  SUBROUTINE TO READ ACTUAL AIRCRAFT PECM MISSION TAPES
                  SUBROUTINE READ (NUM)
  0000000
             SUBROUTINE TO READ EMITTER TARGET AND AIRCRAFT NAVIGATION DATA FROM CARD DATA DECK DATA SEQUENCE MUST BE CF FORMAT TGT/NAV
               COMMON ACLAD(100), ACLAMD(100), ACLAR(100), ACLOD(100), 1ACLOMD(100), ACLOD(100), ALT(100), BRNG(100), BRNGD(100), 2E(100), FREO(1C0), G1(100), G2(100), GATE(100), HDG(100), 3HDGD(100), MODEN(100), MODET(100), NST(100), P11(100), 4P12(100), P22(100), P1TCH(100), PRF(100), PW(100), 5RGLL(100), SLA(100), SLAD(100), SLD(100), SLD(100), 5T(100), TIMEN(100), TIMET(100), TDTD(100), THTD(100), 6THTD1(100), THETA(100), THETAD(100), TLAD(100), TLOD(100), 7VEL(100), VELE(100), VELN(100), XTD(100), YTD(100), 8D11(100), JST(100)
  C
                  DATA PIRAD/57.29578/
  C
                  WRITE(6.53)
DO 1 I=1.100
                  NUM = I - 1
             TYPE=11.0 EMITTER TARGET DATA

READ(5.51.END=2)TGT.TIMET(I),BRNG(I),PRF(I),MODET(I),

1PW(I).FREQ(I)

TYPE=7.0 AIRCRAFT NAVIGATION DATA

READ(5.52) NAV.TIMEN(I).MODEN(I),ALT(I),ACLAR(I),
  C
  C
               1ACLOR(I)
  READ(5.56) HDG(I).VELE(I).VELN(I).ROLL(I).PITCH(I)
  WRITE(6.54) I.TIMET(I).FREQ(I).BRNG(I).PRF(I).PW(I).
                1 MODET(I)
                WRITE(6.55)TIMEN(I), HDG(I), ALT(I), ACLAR(I), ACLOR(I), 1VELN(I), VELE(I), ROLL(I), PITCH(I), MODEN(I)
  CCC
             SUBROUTINE TO COMPUTE AIRCRAFT VELOCITY
                  VELS=VELE(I)空空2+VELN(I)空空2
VEL(I)=SQRT(VELS)
CONTINUE
       1
00002
             SUBROUTINE TO CHANGE ANGLES AND LATITUDE/LONGITUDE FROM RADIANS TO DEGREES + TENTHS OF DEGREES
                  DO 5 J=1.NUM
HDGD(J)=HDG(J)*PIRAD
                  BRNGD(J)=BRNG(J) PIRAD
                  ACLAD(J) = ACLAR(J) * PIRAD
ACLOD(J) = ACLOR(J) * PIRAD
ACLAMD(J) = ACLAD(J)
                   ACLOMD(J) = ACLOD(J)
                  THETA(J)=BRNG(J)+HDG(J)
IF(THETA(J).GT.6.283186) THETA(J)=THETA(J)-6.283186
THETAD(J)=BRNGD(J)+HDGD(J)
IF(THETAD(J).GT.360.0) THETAD(J)=THETAD(J)-360.0
     51
52
53
                   CONT INUE
               FORMAT(7F11.5)
FORMAT(6F13.5)
FORMAT(6F13.5)
FORMAT(11.36X,*LISTING OF EMITTER TARGET DATA AND *,
1'AIRCRAFT NAVIGATION DATA*,//,13X,*TARGET PARAMETERS*
2.37X,*AIRCRAFT PARAMETERS*,//,*TYPE TIME*,4X,
                                                                                                                                                    ALT .
                3 FREQ
                                                                                          MO /
                                          BRNG
                                                               PRF
                                                                               PW
                                                                                                      TIME
                                                                                                                               HDG
```



46X.\*LAT LDNG N.VEL E.VEL ROLL PITCH\*
5.2X.\*MO\*.//)
FORMAT(\* TGT\*.I3.1X.F9.3.1X.F6.1.1X.F7.5.1X.F6.1.1X.
1F4.2.1X.I1)
FORMAT(\* NAV\*.43X.F9.3.1X.F7.5.1X.F7.1.1X.F8.5.1X.F8.5.1X.F8.5.1X.F8.5.1X.F8.3.1X.F8.3.1X.F8.5.X.F8.5.



### BIBLIOGRAPHY

- 1. Pfendtner, F., Adaptive Angle Tracking and Correlation for Airborne Direction Finding, Electrical Engineer Thesis, Naval Postgraduate School, Monterey, Ca., March 1971.
- 2. Coburn, L. L., Kalman Filtering Techniques Applied to Airborne Direction-Finding and Fmitter Location, Aeronautical Engineer Thesis, Naval Postgraduate School, Monterey, Ca., June 1972.
- 3. Kalman, R. E., "A New Approach to Linear Filtering and Prediction Problems," ASME Journal of Basic Engineering, vol. 82, no. 2, pp. 35-45, March 1960.
- 4. Demetry, J. S., Notes on the Theory and Application of Optimal Estimation, paper prepared and copyrighted for use at the Naval Postgraduate School, Monterey, Ca., 1970.
- 5. Athans, M., "The Role and Use of the Stochastic Linear-Quadratic-Gaussian Problem in Control System Design,"

  IEEE Transactions on Automatic Control, vol. AC-16,
  no. 6, pp. 529-552, December 1971.
- 6. Meditch, J. S., Stochastic Optimal Linear Estimation and Control, McGraw-Hill Book Co., Inc., 1969.
- 7. Kayton, M., and Fried, W. R., eds, <u>Avionics Navigation</u>
  Systems, John Wiley and Sons, Inc., 1969.
- 8. United States Naval Security Group Command Reference
  Manual for FXAOSPM Vector Fix Program, Radio Position
  Fixing by Vector Methods, by T. R. McCalla, 30 September 1970.
- 9. Titus, H. A., and Pope, W. R., "Multiple Emitter Airborne Direction-Finding with EOB Utilization," Proc. of the Sixteenth Annual Joint Electronic Warfare Conference, Naval Postgraduate School, Monterey, Ca., December 1970.
- 10. Naval Weapons Center Technical Publication 4652, Application of the Kalman Filter to Aided Inertial Systems, by A. A. Sutherland, Jr., and A. Gelb, 22 August 1968.



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AUTHORIS) (First name, middle initial, last name)

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3. ABSTRACT

A scheme to locate emitter positions using post flight processing of discrete airborne emitter bearing angles-of-arrival information and recorded aircraft position coordinates by Kalman filter techniques is developed. The signal intercept system was assumed to be operating in a multi-emitter environment and all data was sampled at discrete but time varying intervals. The aircraft position data is filtered directly in latitude and longitude and emitter locatoins are computed in latitude and longitude using vector methods. An extended Kalman filtering scheme is developed to compute emitter coordinates directly in latitude and longitude coordinates.



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Thesis

M594 Mills

c.1

Applying the Kalman filter to the emitter location problem using airborne angle-of-arrival information.

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